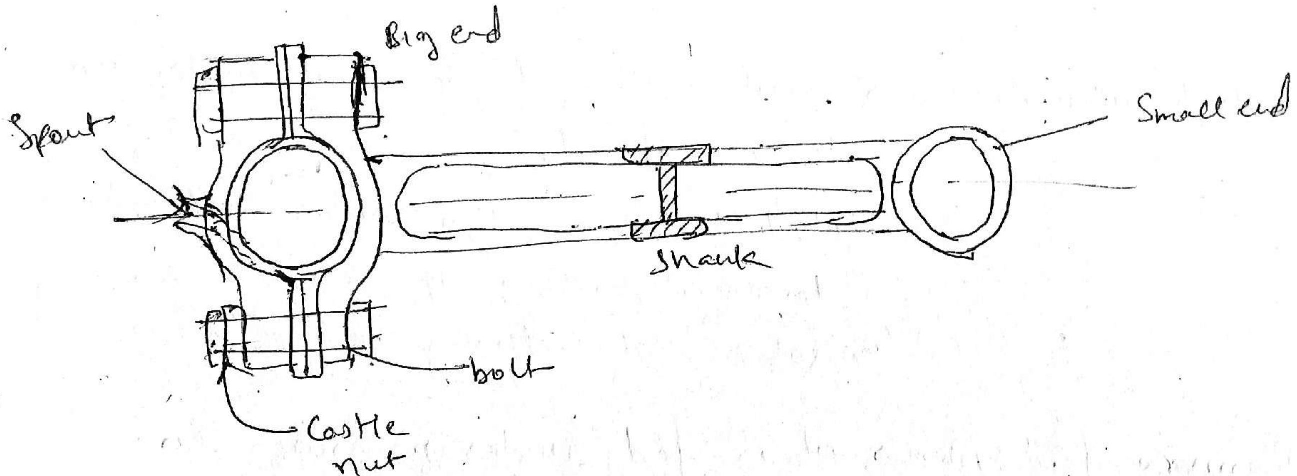
# Unit-II

* Connecting Rod

(1) It consist of an eye to accomodate piston pin.
long shank.
Big end opening split into two parts to accomodate crank pin.




(2) Function — It is used to transmit the push & pull forces from piston pin to crank pin.

(3) Connecting rod ~~transmits~~ converts reciprocating motion to rotary motion

(4) Transmits lubricating oil from crank pin to piston pin.
Provides splash or jet of oil to piston assembly.

(5) manufactured — Drop forging
— outer side unfinished

whole one rod rod forged then the big end is cut into two piece

(6) It is most heavily stressed part. hence alloy steel used.
Subjected to gas force & Inertia.

(7) There are two methods of lubrication of bearings at two ends.

- Splash lubrication
- Pressure lubrication.

Splash lubrication → Spout is attached & set at angle to the axis of rod.

~~splash dips~~ Spout dips into oil during downward motion of connecting rod & splashes oil during upward motion.

Pressure feed system → oil is fed under pressure to the crank pin through the holes drilled in crankshaft.

from big end to small end oil is sent through holes drilled in shank.

(8) Length of connecting rod.

if length is short compared to crank → Connecting rod has greater angular swing hence greater side thrust on piston.

~~if length~~

In high speed engines, $\frac{L}{r}$ is 4 or less.

i.e $\frac{L}{r} \leq 4$

In low speed engines

$\frac{L}{r} = 4$ to $5$

(9) high speed Engines → I-Section.

↓ advantages

(1) less weight & inertia forces

(2) Easy to forge.

low speed Engines → circular C/s is used.

## * Buckling of Connecting Rod

(1) Connecting Rod is slender Engine Component having Considerable length compared to breadth & width.

(2) It is Subjected to compressive ~~stress~~ force

compressive force = Maximum gas pressure load.

Slenderness ratio $= \frac{L}{K}$

Slenderness ratio less than 30 no Buckling

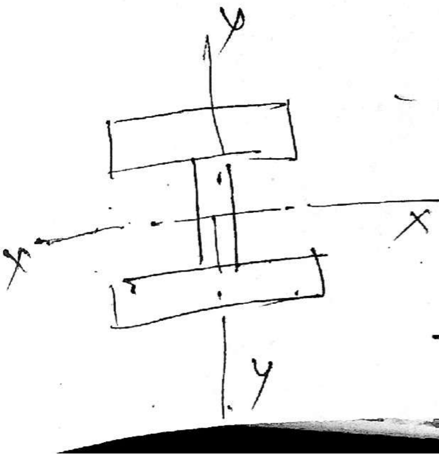
Slenderness ratio more than 30 Buckling occurs

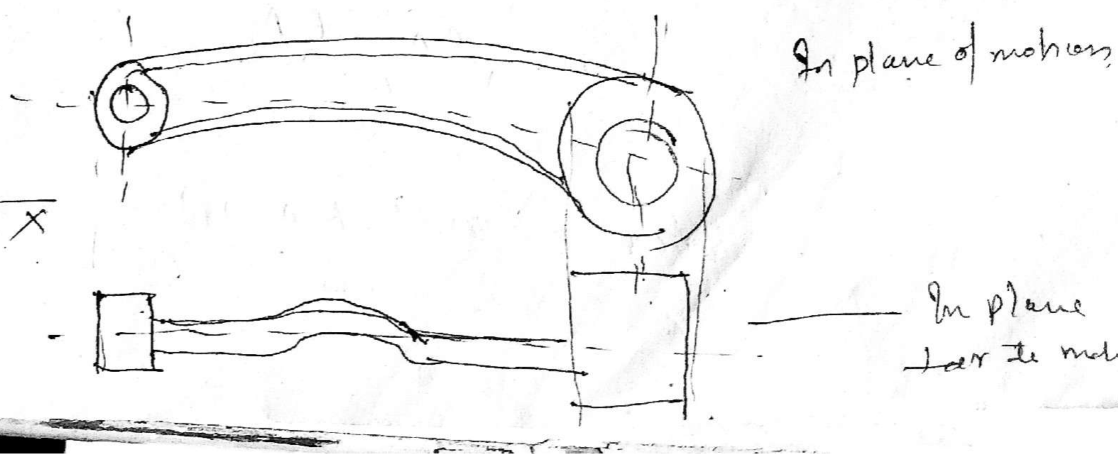
hence Connecting Rod is designed as Column or Strut.

hence Buckling of Connecting rod Can take place in two planes

— In the plane of motion

— In the plane ⊥ar to motion.



(3) ~~In plane x-x~~ the connecting rod is hinged in the plane of motion hence we will consider the end fixity coefficient (n) in Euler's equation as 1.

Euler's equation

$$P_{cr} = \frac{n\pi^2 EA}{(l/k)^2}$$

(4) In the plane ~~x-x~~ the connecting rod the connecting rod is fixed & hence we will consider end fixity coeff. in Euler's equation as 4.

∴ Euler's equation

$$P_{cr} = \frac{n\pi^2 EA}{(l/k)^2}$$

End fixity coeff.

| Sr. No. | End Condition | n |
|---|---|---|
| 1 | Both hinged | 1 |
| 2 | Both fixed | 4 |
| 3 | One fixed one hinged | 2 |
| 4 | One fix. other free | 0.25 |

Where,

$\frac{l}{k}$ = slenderness ratio.

$P_{cr}$ → critical load.

$l$ → length of column

$k$ → radius of gyration of column

$E$ → modulus of elastic

$A$ → Area of C/s.

$n$ = end fixity coeff.

$I$ → Moment of Inertia

$$\therefore \quad k = \sqrt{\frac{I}{A}}$$

So,

$$P_{cr} = \frac{n\pi^2 . EA}{\frac{l^2}{k^2}}$$

$$P_{cr} = \frac{n\pi^2 . EA . k^2}{l^2}$$

$$P_{cr} = \frac{n\pi^2 EA}{l^2} \cdot \frac{I}{A}$$

hence

$$\boxed{P_{cr} \propto I}$$

Therefore, the connecting rod strength depends upon moment of Inertia in buckling.

and hence

(i) for case in plane of motion. i.e along X-X axis

$$n = 1$$

$$P_{cr_{xx}} \propto I_{xx}$$

(ii) for case in ⊥ar to plane of motion. i.e along deflection Y-Y axis

$$n = 4$$

$$P_{cr_{yy}} \propto 4I_{yy}$$

hence the ~~strength along~~ connecting rod is 4 times stronger for buckling along Y-Y axis than in X-X axis

If connecting rod is designed in such a way that it is equally resistant to buckling in both planes then,

$$4I_{yy} = I_{xx}$$

$$4 I_{yy} = I_{xx}$$

$$4 \cancel{A} K_{yy}^2 = \cancel{A} K_{xx}^2$$

$$4 K_{yy}^2 = K_{xx}^2$$

$$\boxed{K_{yy}^2 = \frac{1}{4} K_{xx}^2} \quad \text{--- (1)}$$

Hence for such condition I - section is suitable

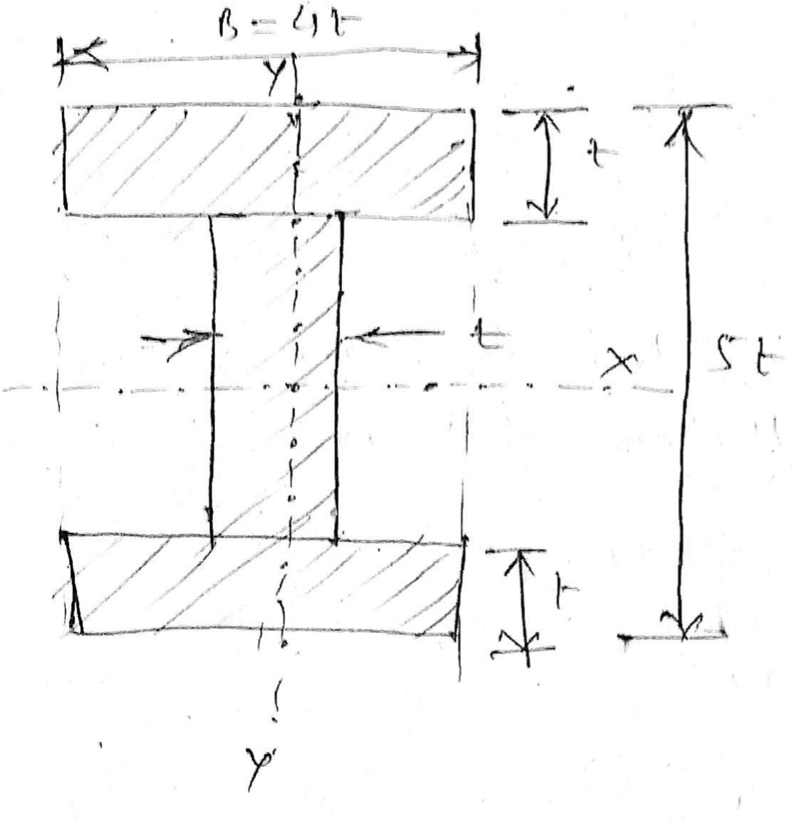



Fig: shows typical c/s of connecting rod.

$$A = 2(4t \times t) + t \times (5t - 2t)$$

$$= 2(4t^2) + t(3t)$$

$$= 8t^2 + 3t^2$$

$$\boxed{A = 11t^2}$$

$$I_{xx} = \left[\frac{1}{12}(4t)(5t)^3\right] - \left[\frac{1}{12}(4t-t)(5t-2t)^3\right]$$

$$= \left[\frac{1}{12}(\cancel{2000t^4}\ 500t^4)\right] - \left[\frac{1}{12}(3t)(3t)^3\right]$$

$$= \left[\frac{500t^4}{12}\right] - \left[\frac{81t^4}{12}\right]$$

$$= \frac{1}{12}\left[500t^4 - 81t^4\right)$$

$$\boxed{I_{xx} = \frac{419}{12}t^4}$$

$$\cancel{K_x}\ I_{xx} = A\,K_{xx}^2$$

$$K_{xx}^2 = \frac{I_{xx}}{A}$$

$$= \frac{419t^{4}}{12} \times \frac{1}{11t^2}$$

$$\boxed{K_{xx}^2 = 3.17t^2} \text{ —— } a$$

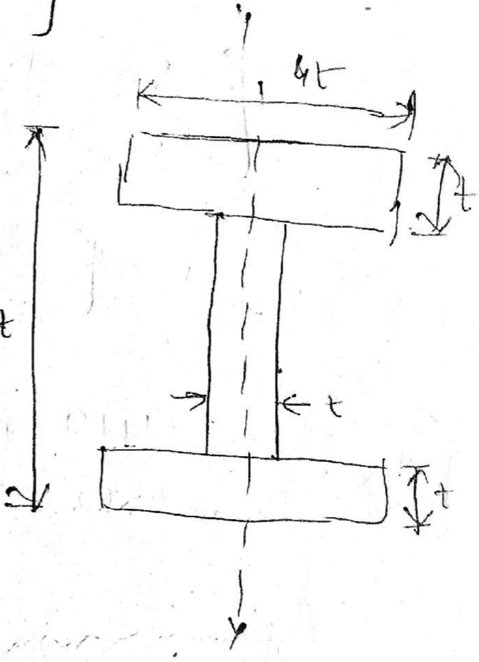
$$K_{xx} = 1.78t$$

$$I_{yy} = 2\left[\frac{1}{12}(t)(4t)^3\right] + \left[\frac{1}{12}(5t-2t)\,t^3\right]$$

$$= 2\left[\frac{64t^4}{12}\right] + \left[\frac{(3t^4)}{12}\right]$$

$$= \frac{128t^4}{12} + \frac{3t^4}{12}$$

$$\boxed{I_{yy} = \frac{131t^4}{12}}$$




$$I_{yy} = A\,k_{yy}^2$$

$$k_{yy}^2 = \frac{I_{yy}}{A}$$

$$= \frac{131\,t^{\cancel{4}\,2}}{12} \times \frac{1}{11\cancel{t^2}}$$

$$\boxed{k_{yy}^2 = 0.99\,t^2} \longleftarrow (b)$$

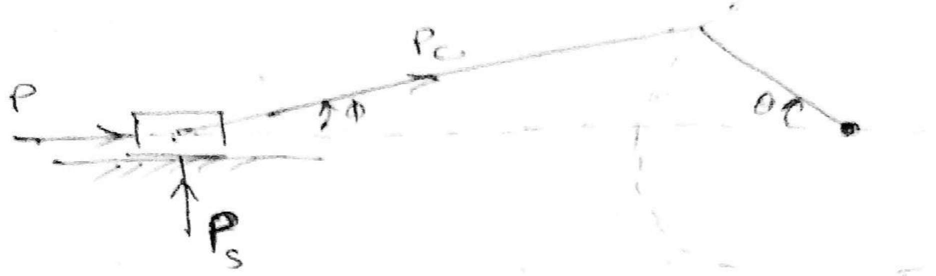
So, here by equation (a) & (b)

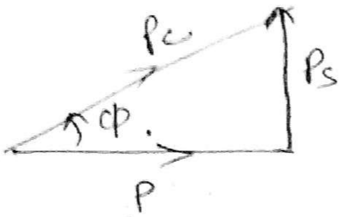
$$\frac{I_{xx}}{I_{yy}} = 3.2$$

$$\boxed{(3.2\,I_{yy}) = I_{xx}} \longleftarrow (2)$$

It is observed that equation (1) & (2) are similar.

## * Cross-section for Connecting Rod




$$P = P_c \cos\phi$$

$$P_c = \frac{P}{\cos\phi}$$

The maximum gas load occurs shortly after dead centre position and at instant $\phi = 3.3$

$$P_c = \frac{P}{\cos 3.3}$$

$$P_c = \frac{P}{0.99}$$

$\boxed{P_c \approx P.}$ — at $\phi = 3.3^\circ$.

hence it can said that force acting on connecting rod is maximum force generated by gas. pressure.

$$P_c = P_{max}\left(\frac{\pi}{4}D^2\right)$$

The I - section is used in connecting rod.

$A = 11t^2$

$K_{xx} = 1.78t$

The dimension can be calculated by Rankine's formula for buckling. In plane of motion.

$$P_{cr} = \frac{\sigma_c A}{1 + a\left(\frac{L}{K_{xx}}\right)^2}$$

$P_{cr}$ = Critical buckling load

$\sigma_c$ = Compressive yield stress.

A = C/s area.

a = Constants depending on mtrl.

L = length of connecting rod.

$K_{xx}$ = Radius of gyration.

$\sigma_c$ = 330 N/mm² (mild steel & plain carbon steel)

$a = \frac{1}{7500}$ (for steel material)

and $P_{cr} = P_c \times (f_s)$

where $f_s$ = 5 to 6

Procedure

(1) Calculate force acting on connecting rod.

$$P_c = \left(\frac{\pi}{4} D^2\right) P_{max}$$

(2) Critical Buckling load

$$P_{cr} = P_c \cdot (f_s)$$ — $f_s$ = 5 to 6.

(3) By Rankine formula

$$P_{cr} = \frac{\sigma_c A}{1 + a\left(\frac{L}{K_{xx}}\right)^2}$$

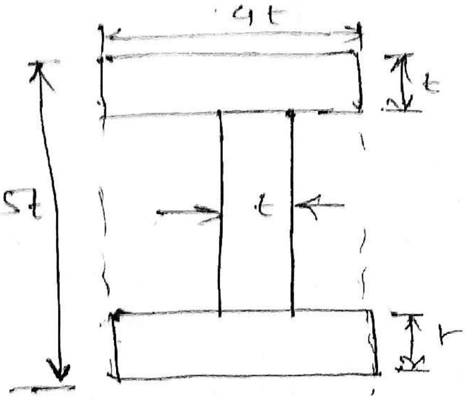
Putting,

$A = 11t^2$, $K_{xx} = 1.78t$, $a = \frac{1}{7500}$, $\sigma_c = 330 N/mm^2$

$\underbrace{a = \frac{1}{7500}}$ Constant depending upon mtrl & end fixity condtn.

hence calculate t

(4) Find dimensions of c/s by using fig



(5) Width is kept constant along the length of connecting rod

(6) Heigh of connecting rod varies

at middle = 5t

at small end = 0.75(5t) to 0.9(5t)

at big end = 1.1(5t) to 1.25(5t)

Q.w

Diesel Engine.

Dimension of connecting rods?

D = 100mm

L = 350mm

$P_{max}$ = 4 Mpa

fs = 6

Ans

$$P_c = P_{max}\left(\frac{\pi}{4}D^2\right)$$

$$= 4\left(\frac{\pi}{4}\right)(100)^2$$

$$\boxed{P_c = 31415.93 N}$$

$P_{cr} = P_c \times f_s$

$= (31415.93)(6)$

$$\boxed{P_{co} = 188495.58\,N}$$

Calculation of t by Rankine form

$$P_{co} = \frac{\sigma_c A}{1 + a\left(\frac{L}{K_{xx}}\right)^2}$$

$$188495.58 = \frac{(330)(11 \cdot t^2)}{1 + \frac{1}{7500}\left(\frac{350}{1.78t}\right)^2}$$

$$\frac{188495.58}{3630} = \frac{t^2}{1 + \frac{5.15}{t^2}}$$

$$(51.92)\left(\frac{t^2 + 5.15}{t^2}\right) = t^2$$

$$(51.92)(t^2 + 5.15) = t^4$$

$$t^4 - 51.92\,t^2 - 267.96 = 0$$

$$t^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t^2 = \frac{51.92 \pm \sqrt{(51.92)^2 - 4(-267.96)}}{2}$$

$$t^2 = \frac{51.92 \pm \sqrt{3767.52}}{2}$$

$$t^2 = \frac{51.92 \pm 61.38}{2}$$

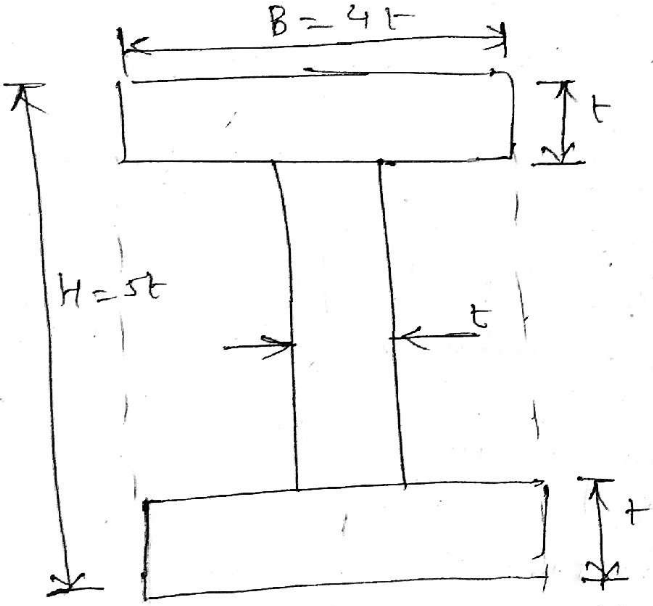
$t^2 = 56.65$ or $-4.73$

$t^2 = 56.65$

$t = 7.53$

$\boxed{t = 8 \text{ mm}}$

Dimension of c/s. :




$H = 5t = 40$

$B = 4t = 32$

height at middle

$H = 5t = 40$

height at small end.

$H_1 = 0.85 H$

$= 34$ mm.

height at big end

$H_2 = 1.2 H$

$= 48$ mm.

* Big & Small End Bearings :-

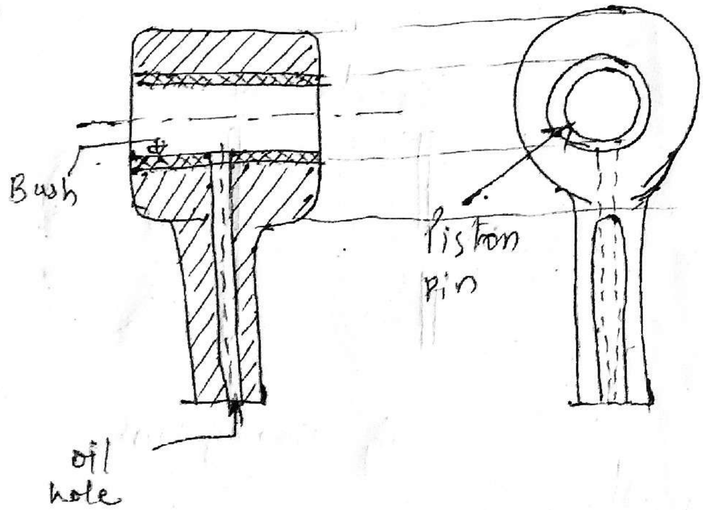



Fig :- Small End of Connecting rod.

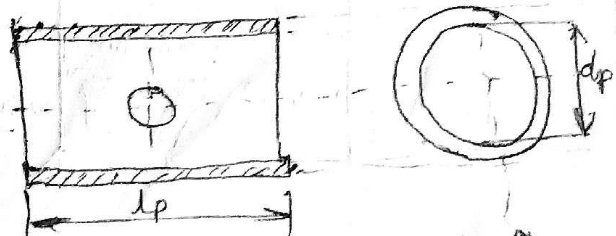



Fig :- Bearing Bush

Material of Piston Pin → Phosphorous Bronze of 3mm thickness
The Bush is one piece solid.
& then grind & reamed.

## Design of Piston Pin Bush

It is designed by bearing consideration.

$$P_c = (P_{max})\left(\frac{\pi}{4} D^2\right)$$

$$\boxed{P_c = d_p\, l_p\, (P_b)_p}$$

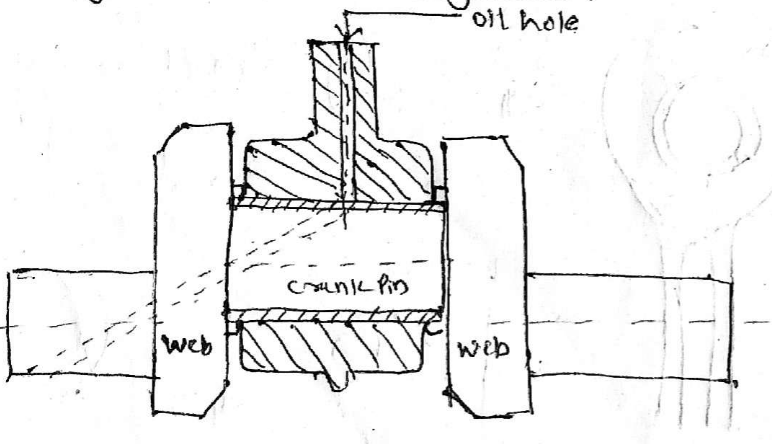
where,

$d_p$ = dia of Piston Pin or Inner dia. of bush
$l_p$ = length of bush on piston Pin
$(P_b)_p$ = allowable bearing pressure for the Piston pin bush.
= 10 to 12.5 Mpa.

and

$$\left(\frac{l_p}{d_p}\right) = 1.5 \text{ to } 2.$$

## Big-End of Connecting Rod



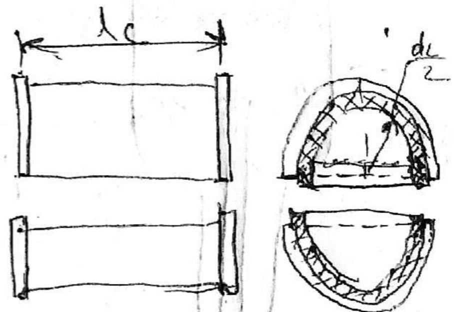
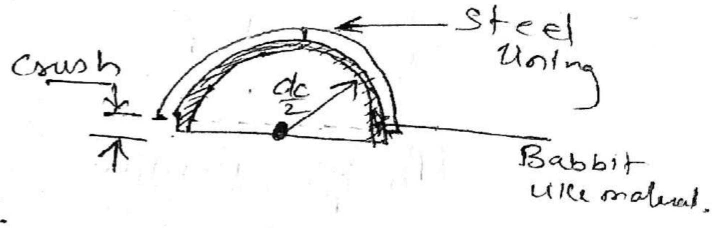




Fig: Bearing bush.

The crank pin bearing is split into two halves. lined bushing consists of steel backing with a thin lining of bearing material like babit.

It is also designed for bearing consideration




$$\boxed{P_c = d_c \cdot l_c \cdot (P_b)_c}$$

where,

$d_c$ = dia. of crank pin or inner dia. of bush of crank pin

$l_c$ = length of crank pin or length of bush of crank pin.

$(P_b)_c$ = allowable bearing pressure for crank pin bush.

= 5 to 10 N/mm²

$$\frac{l_c}{d_c} = 1.25 \text{ to } 1.5.$$

Crush → when cap is tightened by bolts, the projecting bearing faces are squeezed in to form press fit between the split bushes. & cap.

Shim → The wear of big end bearing is compensated by means of thin metallic strip between the cap and fixed half.

As the wear takes place one or more thin sheet are removed & cap is tightened.

25.13 Determine the dimes of Small end & big end bearing of connecting rod.

$D = 100\text{ mm}$

$P_{max} = 4\text{ Mpa}$

$\left(\frac{l_p}{d_p}\right) = 2$

$\left(\frac{l_c}{d_c}\right) = 1.3$

$(P_b)_p = 12\text{ Mpa}$

$(P_b)_c = 7.5\text{ Mpa}.$

I maximum bearing load.

$P_c = (P_{max})\left(\frac{\pi}{4}D^2\right)$

$= (4)\left[\frac{\pi}{4}100^2\right]$

$\boxed{P_c = 31415.93\text{ N}}$

Piston Pin bearing

$P_c = d_p\, l_p\, (P_b)_p.$

$31415.93 = d_p(2d_p)(12)$

$31415.93 = 24\, d_p^2$

$d_p^2 = 1308.99$

$\boxed{d_p = 36.18\text{ mm}}$

~~Crank Pin~~

$l_p = 2d_p$

$\boxed{l_p = 72.36\text{ mm}.}$

Crank Pin bearing

$P_c = d_c\, l_c\, (P_b)_c$

$31415.93 = (d_c)(1.3 d_c)(7.5)$

$31415.93 = 9.75\, d_c^2$

$d_c^2 = 3222.14$

$\boxed{d_c = 56.76\text{ mm}}$

$l_c = 1.3\, d_c$

$\boxed{l_c = 73.79\text{ mm}}$

* Big End Cap and Bolts

The maximum force acting on the cap and the bolts consists only of inertia force at T.D.C. on the exhaust stroke.

$$P_i = m_r \omega^2 r \left[ \cos\theta + \frac{\cos 2\theta}{n_1} \right]$$

$P_i$ = Inertia force on the cap or bolts.

$m_r$ = mass of reciprocating parts

$= m_p + \frac{1}{3} m_c$ (By two mass system)

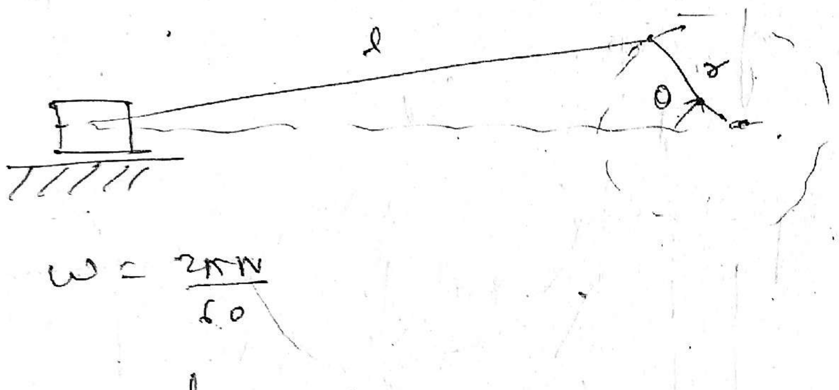
$\omega$ = Angular velocity of crank

$r$ = crank radius

$n_1$ = obliquity ratio $= \left(\frac{L}{r}\right)$

$L$ = length of connecting rod

$r$ = radius of crank



$$\omega = \frac{2\pi N}{60}$$

$$r = \frac{l}{2}$$

maximum inertia will be when

$\cos\theta = 1$ or $\cos 2\theta = 1$

at $\theta = 0^\circ$ so, ~~when~~ the inertia will be maximum & its value will be,

$$\boxed{(P_i)_{max} = m_r \omega^2 . r \left[ 1 + \frac{1}{n_1} \right]}$$

Similarly the maximum force load will be taken by bolts

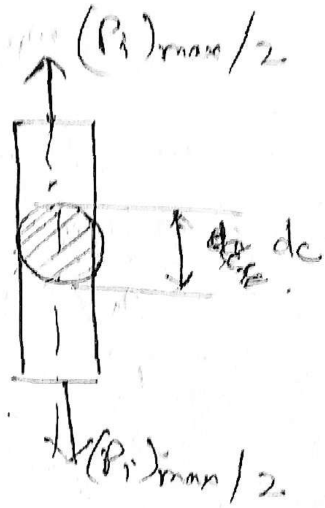
$$(P_i)_{max} = 2\left(\frac{\pi}{4} d_c^2\right) \cdot \sigma_t$$

here

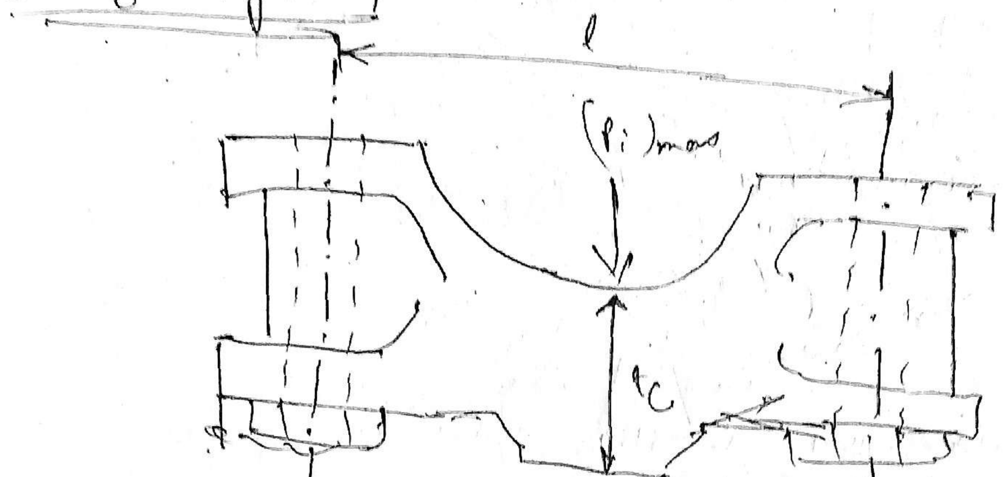
$d_c$ = Core diameter of bolts.

$\sigma_t$ = Permissible tensile stress for bolts material.

Nominal dia. of bolts, $d = \left(\frac{d_c}{0.8}\right)$




Design of Cap :-




① The Cap is also called Keep plate.

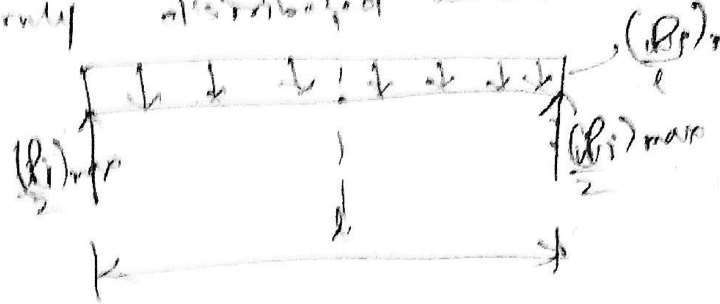
② It is Subjected to inertia force $(P_i)_{max}$.

③ It is treated as beam freely supported at the bolt centres & loaded in intermediate between

— uniformly distributed load

— ~~UDL~~ Concentrated load at Centre

— uniformly distributed load :




B.M @ centre ,

$$= \left[\frac{(P_i)_{max}}{2} \times \frac{l}{2}\right] - \left[\frac{(P_i)_{max}}{l} \times \frac{l}{2} \times \frac{l}{4}\right]$$

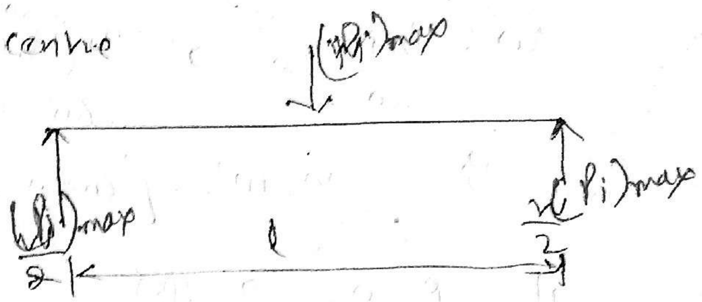
$$= \left[\frac{(P_i)_{max} l}{4}\right] - \left[\frac{(P_i)_{max} l}{8}\right]$$

$$= -\frac{(P_i)_{max} l}{8} + \frac{(P_i)_{max} l}{4}$$

B.M at centre $= \frac{(P_i)_{max} l}{8}$

— Concentrated load @ centre

B.M @ Centre




B.M @ Centre $= \left[\frac{(P_i)_{max}}{2} \times \frac{l}{2}\right]$

$$= \frac{(P_i)_{max} l}{4}$$

Hence we take intermediate of this two value

$$\boxed{M_b = \frac{(P_i) l}{6}}$$

where $l = d_c + 2$ (thickness of bush) + (nominal dia of bolt) + clearence (3 mm)

so, $\boxed{M_b = \frac{(P_i)_{max} l}{6}}$

Bending stress,

$$\boxed{\sigma_b = \frac{M_b y}{I}}$$

where $y = \left(\frac{t_c}{2}\right)$ , $I = \frac{b_c t_c^3}{12}$.

$b_c$ = width of cap = length of crank pin $= l_c$

25.14

$N = 1800 \, rpm$

$L = 350 \, mm$

length of stroke $= 175 = 2r$

$r = \frac{175}{2} = 87.5 \, mm$

$m_r = 2.5 \, kg$

$l_c = 76 \, mm$

$d_c = 58 \, mm$

thickness of bush $= 3 \, mm$.

bolts, $\sigma_t = 60 \, N/mm^2$

Cap, $\sigma_b = 80 \, N/mm^2$

Nominal dia, $d = ?$

Cap $t_c = ?$

$n_1 = \frac{L}{r}$

$n_1 = \frac{350}{87.5} = 4.$

A.

$\omega = \frac{2\pi N}{60} \Rightarrow \frac{2\pi(1800)}{60} \Rightarrow 188.49 \, rad/sec.$

$P_i = m_r \omega^2 . r \left[ \cos\theta + \frac{\cos 2\theta}{n_1} \right]$

at $\theta = 0$, $P_i = (P_i)_{max}$

$(P_i)_{max} = m_r r . \omega^2 \left[ 1 + \frac{1}{n_1} \right]$

$(P_i)_{max} = (2.5)(\cancel{87.5}\ 0.0875)(188.49)^2 \left[ 1 + \frac{1}{4} \right]$

$(P_i)_{max} = 7771.855 \left[ \frac{5}{4} \right]$

$\boxed{(P_i)_{max} = 9714.818.77 \, N}$

Design of bolts,

$(P_i)_{max} = 2\left( \frac{\pi}{4} . d_c^2 \right) \sigma_t$

$9714818.77 = \cancel{2}\left( \frac{\pi}{\cancel{4}\,2} d_c^2 \right)(60)$

$$\boxed{d_c = 10.15\,mm}$$

$$d = \left(\frac{d_c}{0.8}\right) = 12.69$$

$$\boxed{d \simeq 16\,mm}$$

$b_c = l_c = 76\,mm$

$l = d_c + 2$(thickness of bush) + (bolt dia.) + clearence

$$l = 58 + 2(3) + 16 + 3$$

$$\boxed{l = 83\,mm}$$

Q Bending stress in cap.

$$\sigma_b = \frac{M_b y}{I}$$

$$80 = \frac{\left(\frac{(P_i)_{max}\, l}{6}\right)\left(\frac{t_c}{2}\right)}{\left(\frac{b_c \cdot t_c^3}{12}\right)}$$

$$80 = \frac{\left(\frac{9714.81 \times 83}{6}\right)\left(\frac{t_c}{2}\right)}{\left(\frac{76 \cdot t_c^3}{12}\right)}$$

$$\frac{80 \times 76 \times t_c^2}{\cancel{12}} = \frac{9714.81 \times 83}{\cancel{12}}$$

$$t_c^2 = 132.61$$

$$t_c = 11.51\,mm \simeq 12\,mm$$

$$\boxed{t_c = 12\,mm}$$

* **Whipping stress** {whipping → चाबकाचे मारणे.}

Small end of connecting rod → translation motion

big end — " — " → rotational motion.

Intermediate point — " — " → elliptical orbital motion

The lateral oscillation of connecting rod induces inertia forces that acts all along the connecting rod. causing Bending.

lateral oscillation → due to inertia forces → Bending of connecting rod.

~~The bending~~ This action is called whipping.

mass of connecting rod, per metre length.

$m_1$ = volume × density

$m_1$ = area × length × density

$m_1 = A(1)(\rho)$

$$\boxed{m_1 = A\rho}$$

For steels,

$\rho = 7800\ kg/m^3$

~~$m_1 = 11t^2$ — for I section.~~

$m_1 = (11t^2)\rho$.

∴ Maximum bending moment occurs at $\frac{L}{\sqrt3}$ from Piston and its magnitude is

$$(M_b)_{max} = m r\omega^2 \cdot \frac{L}{9\sqrt3}$$

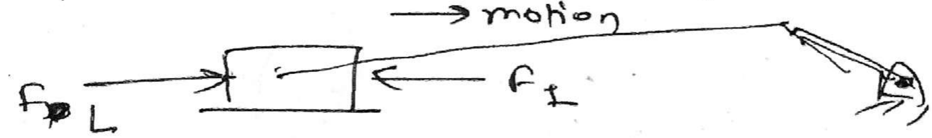
∴ $m = m_1 L$ — per length data

$$\boxed{(M_b)_{max} = m_1 r\omega^2 \frac{L^2}{9\sqrt3}}$$

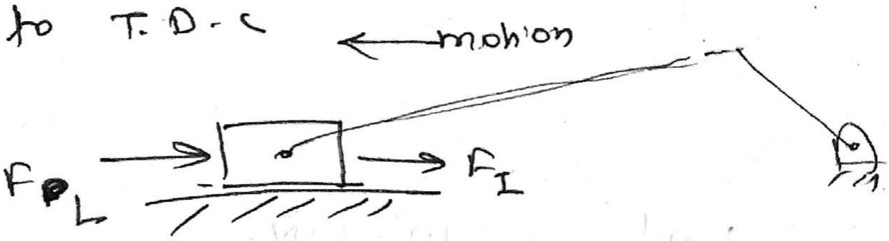
$I_{xx} = \left(\frac{419}{12}\right)t^4$, $y = \left(\frac{5t}{2}\right)$, $\sigma_b = \frac{M_b y}{I}$

$$F_I = m_R \omega^2 \cdot R\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

Inertia force of reciprocating parts opposes the force on piston. when it moves from T.D.C to B.D.C.




Inertia force ~~on~~ of ~~rec~~ reciprocating parts help the force on piston when it moves from B.DC to T.D.C




So,

$$F_P = F_L \pm F_I$$

where

$F_P$ = force on reciprocating body (Net force)

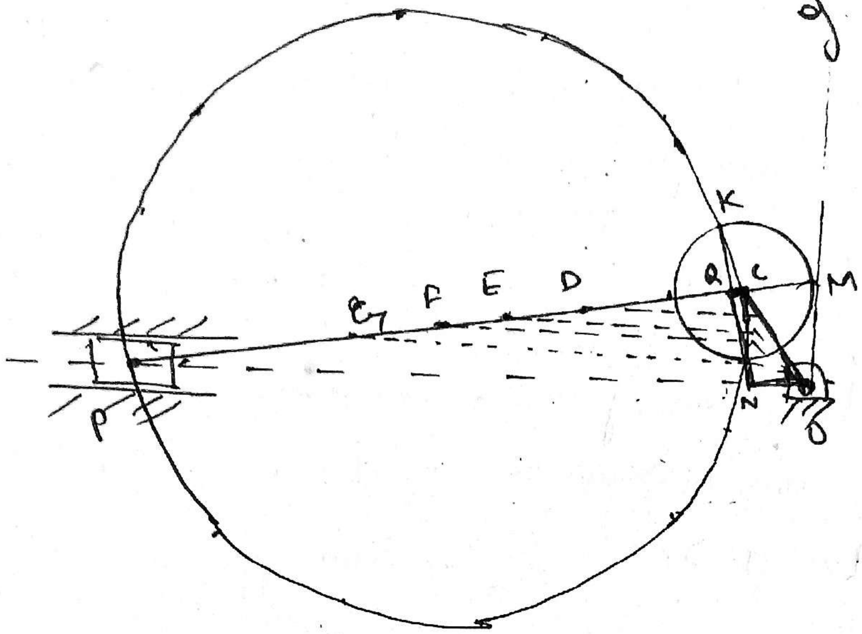
$F_L$ = —"— due to gas pressure

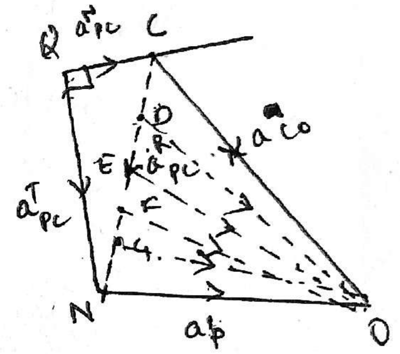
$F_I$ = Inertia force.

when weight is considered then.

$$F_P = F_L + \cancel{\pm} F_I \pm W_R$$

By Klein's construction we can find acceleration of connecting rod.





$a^T_{PC} = \omega^2 QN.$

~~Inertia~~ Similarly we can find acceleration at G, F, E, D. by drawing parallel from the resultant acceleration of connecting rod.

acceleration at $C = \omega^2 CO$.

—"— at $G = \omega^2 GO$.

—"— at $F = \omega^2 FO$

and so on.

Inertia forces will be (Inertia = mass x accelerat.)

Inertia ~~at~~ force at $C = m \times \omega^2 CO$

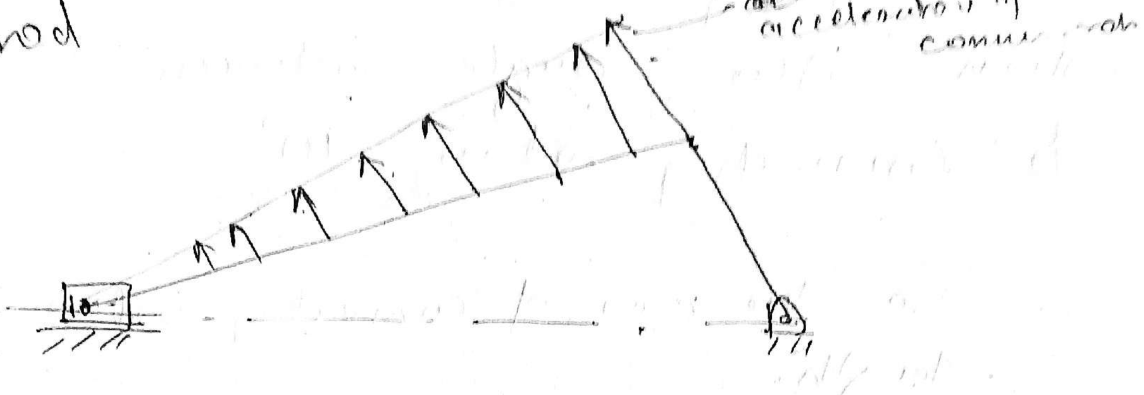
—"— —"— at $D = m\omega^2 DO$

—"— —"— —"— $E = m\omega^2 EO$

and so on.

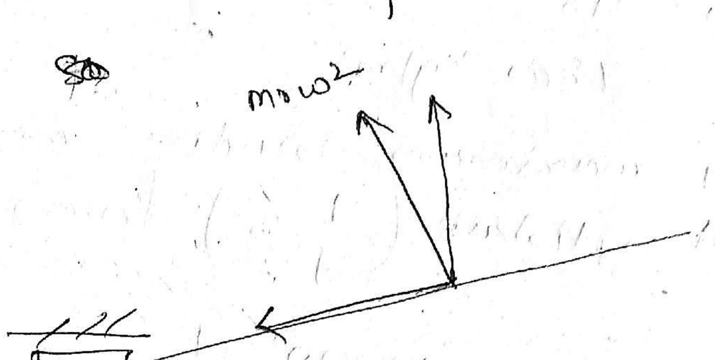
Note on whipping stress.

By Klein's Construction we can find the acceleration of connecting rod




acceleration of connecting rod will go decreasing toward piston

The inertia force will be in ~~direction~~ the same axis of acceleration.



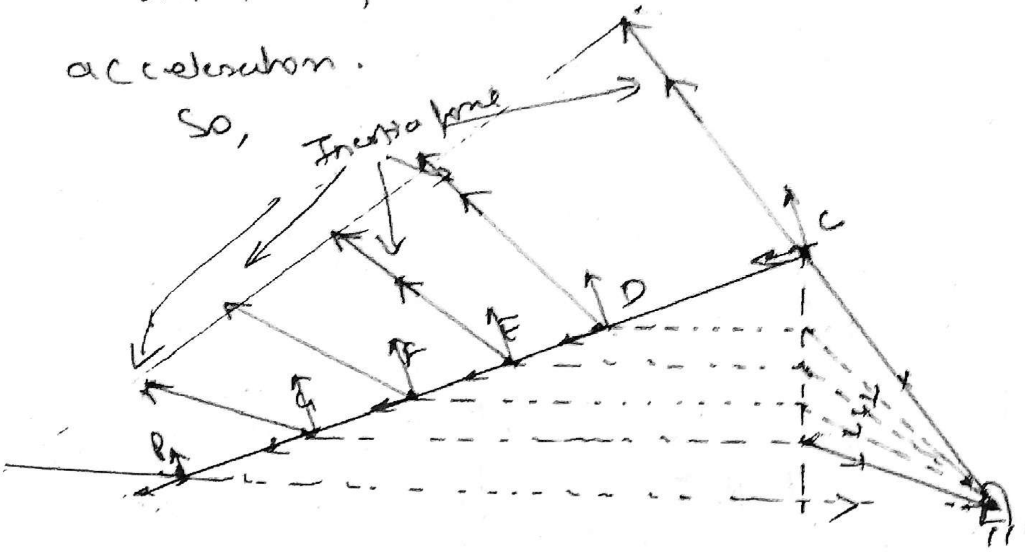

So, the inertia force can be resolved ~~and~~ in two components

— One horizontal (causing whipping stress)
or ⊥ ar to Connecting rod

— One vertical
along the axis of connecting rod. (We have considered)

Inertia force will be in opposite direction acceleration.

So,

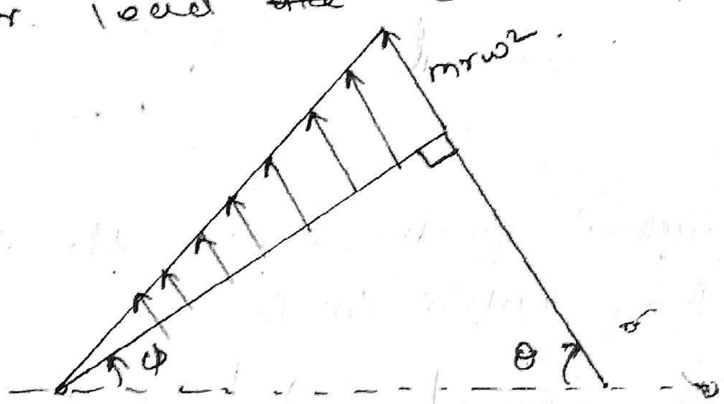



Inertia force can resolved into two component Parallel components & ⊥ar components

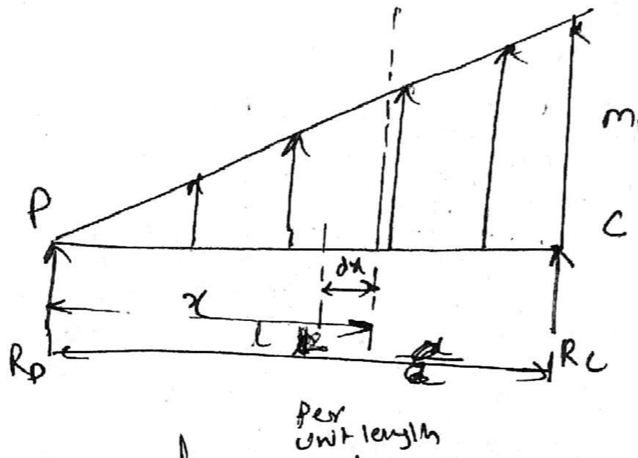
Parallel components → It adds up to all together to the force on connecting rod.

⊥ar component → This produces bending action called whipping stress.

* ~~⊥ar~~ component will be maximum when angle between connecting rod & crank is 90°. In that ~~case~~ case variation on connecting rod by inertia force will be linear load ~~like~~ on simply supported beam.



Assume connecting rod of uniform ~~the~~ c/s. and mass $m_1$ kg per unit length.




① Inertia force at crank pin $= m_1 \cdot r \cdot \omega^2$

② —— " —— " — " — Piston pin $= 0$

③. Inertia force due to small element of length $dx$ at distance $x$ from piston pin

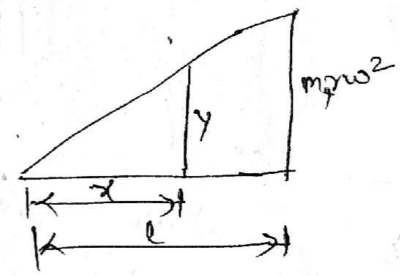
$$dF_I = m_1 \cdot r \cdot \omega^2 \cdot \frac{x}{l} \cdot dx$$

$$F_I = \int_0^l m_1 \cdot \omega^2 \cdot r \cdot \frac{x}{l} \cdot dx$$

$$F_I = \frac{m_1 r\omega^2}{l} \left[\frac{x^2}{2}\right]_0^l$$

$$F_I = \frac{m_1 r\omega^2}{l} \cdot \frac{l^2}{2}$$

$$\boxed{F_I = \frac{m_1 r\omega^2 \cdot l}{2}}$$



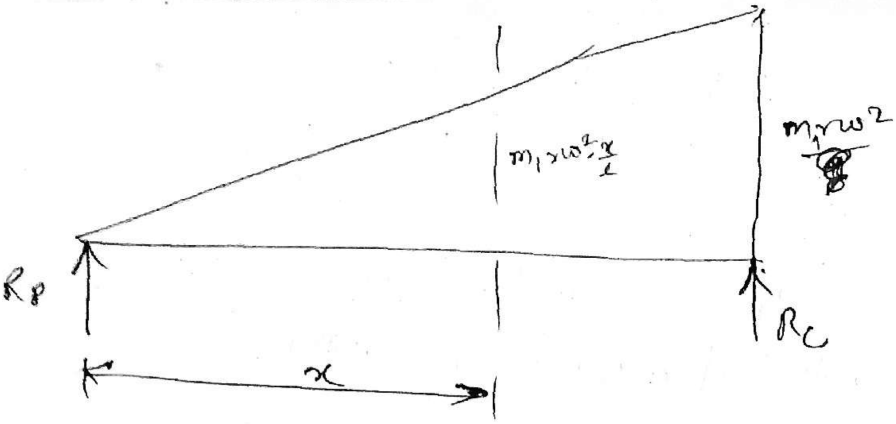

$$\frac{y}{x} = \frac{m_1 r\omega^2}{l}$$

$$y = \frac{m_1 r\omega^2}{l} \cdot x$$

This resultant inertia force acts at a distance of $\frac{2 \times l}{3}$ from piston pin P.

Since it assumed $\left(\frac{1}{3}\right)$rd mass of connecting rod is concentrated at Piston pin P and $\left(\frac{2}{3}\right)$rd at crank pin.

word at ...




$\frac{m_1 r \omega^2}{l} = \frac{y}{x}$

$y = \frac{m_1 r \omega^2 \cdot x}{l}$.

$$M_x = \left\{ (R_P \cdot x) \right\} - \left\{ \left[ \frac{1}{2} \times \left( m_1 r \cdot \omega^2 \cdot \frac{x}{l} \right) \times x \right] \times \frac{x}{3} \right\}$$

$\therefore \quad R_P = \frac{F_I}{3}$ Dividing & multiplying by $l$

$$M_x = \left\{ \frac{F_I}{3} \cdot x \right\} - \left\{ \left( \frac{1}{2} \times m_1 r \cdot \omega^2 l \right) \frac{x^2}{l^2} \cdot \frac{x}{3} \right\}$$

$$= \frac{F_I x}{3} - \frac{F_I \cdot x^3}{3 l^2}$$

$$\boxed{M_x = \frac{F_I}{3} \left[ x - \frac{x^3}{l^2} \right]}$$

for maximum B.M. different w.r.t $x$.

$$\frac{dM_x}{dx} = \frac{F_I}{3} \left[ 1 - \frac{3x^2}{l^2} \right]$$

To get maxima value $\frac{dM_x}{dx} = 0$

$$\frac{F_I}{3} \left[ 1 - \frac{3x^2}{l^2} \right] = 0$$

$$1 = \frac{3x^2}{l^2}$$

$$x = \sqrt{\frac{l^2}{3}}$$

$$\boxed{x = \frac{l}{\sqrt{3}}}$$

Substituting this value in $M_x$ to get magnitude of maximum B.M.

$$(M_x)_{max} = \frac{F_I}{3}\left[x - \frac{x^3}{l^2}\right]$$

$$(M_x)_{max} = \frac{F_I}{3}\left[\frac{l}{\sqrt{3}} - \frac{l^3}{\cancel{l^2} \cdot 3\sqrt{3}}\right]$$

$$(M_x)_{max} = \frac{F_I}{3}\left[\frac{l}{\sqrt{3}} - \frac{l}{3\sqrt{3}}\right]$$

$$(M_x)_{max} = \frac{F_I l}{3}\left[\frac{1}{\sqrt{3}} - \frac{1}{3\sqrt{3}}\right]$$

$3 \mid 3, 3\sqrt{3}$
$\sqrt{3} \mid 1, \sqrt{3}$
$\quad 1, \sqrt{3}$

$$(M_x)_{max} = \frac{F_I \cdot l}{3}\left[\frac{3-1}{3\sqrt{3}}\right]$$

$$(M_x)_{max} = \frac{F_I \cdot l\,(2)}{9\sqrt{3}}$$

$$(M_x)_{max} = \frac{2F_I \cdot l}{9\sqrt{3}}$$

$$(M_x)_{max} = \cancel{2} \cdot \frac{(m_1 r \omega^2 \cdot l)}{\cancel{2}} \cdot \frac{l}{9\sqrt{3}}$$

$$\boxed{(M_x)_{max} = \frac{m_1 r \omega^2 l^2}{9\sqrt{3}}}$$

$$y = \left(\frac{5t}{2}\right)$$

$$I_{xx} = \left(\frac{419.}{12}\right)t^4$$

$$\boxed{\sigma_b = \frac{(M_{x})_{max} \cdot y}{I}}$$

2.5.14 Data for Cap & bolts of big end connecting rod.

$N = 1800$ rpm

$l = 350$ mm

length of stroke : $2r = 175$

$r = \frac{175}{2} = 87.5$ mm.

$m_r = 2.5$ kg

$l_c = 76$ mm

$d_c = 58$ mm

thickness of bush = 3mm

$(\sigma_t)_{bolts} = 60$ N/mm²

$(\sigma_b)_{cap} = 80$ N/mm²

$d = ?$

$t_c = ?$

Sol.

$n_1 = \frac{l}{r} = \frac{350}{87.5} = 4$ , $\omega = \frac{2\pi N}{60} = 188.5$ rad/s

$(P_1)_{max} = m_r r \omega^2 \left[1 + \frac{1}{n_1}\right]$

$= (2.5)(87.5)(188.5)^2\left[1 + \frac{1}{4}\right]$

$\boxed{(P_1)_{max} = 9715.85 \text{ N}}$

~~$(P_1)_{max} = 2\left(\frac{\pi}{4} d_c^2\right)\sigma_t$~~ $\sigma_t = \frac{(P_1)_{max}}{2\left(\frac{\pi}{4} d_c^2\right)}$

$$\boxed{d_c = 10.15\,mm}$$

$$d = \frac{d_c}{0.8}$$

$$\boxed{d = 12.69\,mm}$$

$$\boxed{d = 16\,mm}$$

Thickness of cap

$b_c = l_c = 76\,mm.$

$l$ = dia of crank pin + thickness of bush + normal dia of bolt + clea...

$= 58 + 2(3) + 16 + 3$

$$\boxed{l = 83\,mm}$$

$$M_b = \frac{(P_t)_{max} \cdot l}{6} = 134407.59\,N\cdot mm$$

$$I = \frac{b_c\, t_c^3}{12} = 6.33\, t_c^3$$

$$y = \frac{t_c}{2}$$

$$\sigma_b = \frac{M_b\, y}{I}$$

$$\boxed{t_c = 12\,mm}$$

25.15

$N = 1800\,rpm$

$l = 350\,mm$

$r = \frac{175}{2} = 87.5$

$n_1 = \frac{l}{r} = 4.$

$\rho = 7800\,kg/m^3$

$t = 8\,mm$

$A = 11t^2,\quad I_{xx} = \left(\frac{419}{12}\right)t^4 \quad \& \quad y = \left(\frac{5t}{2}\right)$

Whipping stress = ?

$$\omega = \frac{2\pi N}{60} = \frac{2\pi(1800)}{60} = 188.5 \text{ rad/se.}$$

$$(M_b)_{max} = m_1 \cdot r\,\omega^2 \frac{L^2}{9\sqrt{3}}$$

$$\therefore \quad m_1 = (11t^2)\rho$$

$$= \left[11\,(0.008)^2\right] 7800$$

$$\boxed{m_1 = 5.49 \text{ kg/m}}$$

$$(M_b)_{max} = m_1 r\,\omega^2 \left(\frac{L^2}{9\sqrt{3}}\right)$$

$$= (5.49)(0.0875)(188.5)^2\left[\frac{(0.35)^2}{9\sqrt{3}}\right]$$

$$= 134.13 \text{ N-m}$$

$$\boxed{(M_b)_{max} = 134.13 \times 10^3 \text{ N-mm}}$$

$$\sigma_b = \frac{M_b y}{I}$$

$$= \frac{134.13 \times 10^3 \times \left(\frac{5(8)}{2}\right)}{\left(\frac{419}{12}\right) t^4}$$

$$\boxed{\sigma_b = 18.76 \text{ N/mm}^2}$$

Q5.10

$D = 85 \, mm$

$l = 350 \, mm$

$P_{max} = 3 \, Mpa$

$(f_s)_{buckling \, factor} = 5$

$\frac{l_p}{d_p} = 1.5$

$\frac{l_c}{d_c} = 1.25$

$(P_b)_p = 13 \, Mpa$

$(P_b)_c = 11 \, Mpa$

$2r = 140$

$r = 70$ $\quad n_1 = \frac{l}{r} = \frac{350}{70} = 5$

$m_r = 1.5 \, kg$

$N = 2000 \, rpm$ $\quad \omega = \frac{2\pi N}{60} = 209.44 \, rad/sec$

thickness of bush = 3 mm

Material of cap = Steel 40C8

$(\sigma_{yt})_{cap} = 380 \, N/mm^2$

$(f_s)_{cap} = 4$

Material of bolt = Chromium molybdenum

$(\sigma_{yt})_{bolt} = 450 \, N/mm^2$

$(f_s)_{bolt} = 5$

$\rho = 7800 \, kg/m^3$

(i) Dimension of c/s of connecting rod

(ii) --- of big end & small end.

(iii) $d = ?$

(iv) $t_c = ?$

(v) magnitude of whipping stress

A.

~~Pmax~~

$P_C = P_{max}\left(\frac{\pi}{4}D^2\right)$

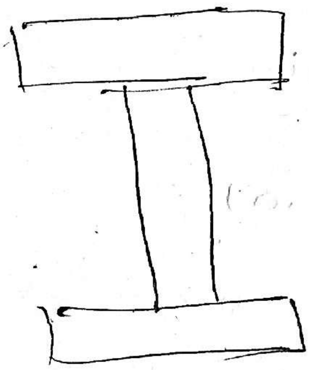
$P_c = (3)\left(\frac{\pi}{4} \times 85^2\right)$

$\boxed{P_c = 17023.51N}$

$P_{cr} = P_c(fs)$

$= (17023.51)(5)$

$\boxed{P_{cr} = 85117.55N}$



$A = 11t^2$

~~$I_{xx}$~~

$K_{xx} = 1.78t$

$a = \frac{1}{7500}$

$\sigma_c = 330N/mm^2$

using rankine formula.

$$P_{cr} = \frac{\sigma_c A}{1 + a\left(\frac{L}{K_{xx}}\right)^2}$$

$$85117.55 = \frac{(330)(11t^2)}{1 + \frac{1}{7500}\left(\frac{350}{1.78t}\right)^2}$$

$$(85117.55)\left[1 + \frac{5.15}{t^2}\right] = 3630t^2$$

$$8517.55\,t^2 + 438355.38 = 3630\,t^4$$

$$3630\,t^4 - 85117.55\,t^2 - 438355.38 = 0$$

$$t^4 - 23.44\,t^2 - 121 = 0$$

$$t^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t^2 = \frac{23.44 \pm \sqrt{(23.44)^2 - 4(1)(-121)}}{2}$$

$$t^2 = \frac{23.44 \pm \sqrt{1033.43}}{2}$$

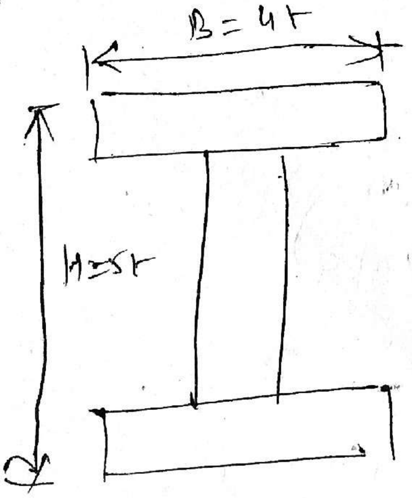
$$t^2 = \frac{23.44 \pm 32.14}{2}$$

$$t^2 = 11.72 \pm 16.07$$

$t^2 = 27.80$ or -ve value

$t = 5.27$ mm

$$\boxed{t = 5.5\text{ mm}}$$




$B = 4(t) = 4(5.5)$

$= 22$ mm

$H = 5(t) = 5(5.5) = 27.5$ mm

thickness of web $= 5.5$ mm

" " flanges $= 5.5$ mm.

Variation in height

at middle section, $H = 5t = 27.5$ mm

at small end, $H_1 = 0.85H = 0.85(27.5)$

$= 23.38$

$\approx 24$ mm

at big end $H_2 = 1.2H$

$= 1.2(27.5)$

$= 33$ mm

(iii) Dimensions of small end & big end Bearing:

(a) Piston pin bearing,

$P_c = (P_b)_p \cdot d_p \cdot l_p$

$17023.51 = (13)(d_p)(1.5 d_p)$

$17023.51 = 19.5 d_p^2$

$d_p^2 = 873.005$

$d_p = 29.54$ mm

$\boxed{d_p = 30 \text{ mm}}$

$l_p = d_p \cdot 1.5$

$\boxed{l_p = 45 \text{ mm}}$

(b) ~~Cast~~ Crank Pin bearing

$$P_c = (P_b)_c \, d_c \cdot l_c$$

$$17023.51 = (11)(d_c)(1.25 d_c)$$

$$17023.51 = 13.75 \, d_c^2$$

$$d_c^2 = 1265.68$$

$$d_c = 35.57$$

$$\boxed{d_c = 36 \text{ mm}}$$

$$l_c = 1.25 \, d_c$$

$$l_c = 1.25 (36)$$

$$\boxed{l_c = 45 \text{ mm.}}$$

(iii) bolts,

Inertia force of connecting rod

$$(P_1)_{max} = m_r r \omega^2 \left[1 + \frac{1}{n_1}\right]$$

$$(P_1)_{max} = (1.5)(0.070)(209.44)^2\left[1 + \frac{1}{5}\right]$$

$$\boxed{(P_1)_{max} = 5527.00 \text{ N}}$$

Design of bolts

$$(\sigma_t)_{bolt} = \frac{(P_1)_{max}}{2\left(\frac{\pi}{4} \cdot d_c^2\right)}$$

$$450 = \frac{5527 \times 4}{\pi \times d_c^2}$$

$$d_c^2 = 39.1$$

$$d_c = 6.25 \text{ mm}$$

$$d = \frac{d_c}{0.8}$$

$$d = \frac{6.25}{0.8}$$

$$d = 7.81 \text{ mm}$$

$$\boxed{d = 8 \text{ mm}}$$

(iv) Thickness of cap,

$$\sigma_b = \frac{M_b y}{I}$$

$$M_b = \frac{Pl}{6}$$

$$= \frac{(5527)(53)}{6}$$

$$\boxed{M_b = 48821.83 \text{ N-mm}}$$

l = dia of crank pin (d_0) + nom. dia of bolts (d) + 2 (thickness of bush) + 6 mm

$$l = 36 + 8 + 2(3) + 3$$

$$\boxed{l = 53 \text{ mm}}$$

$$b_c = l_c = 45 \text{ mm}$$

$$\sigma_b = \frac{(48821.83)\left(\frac{t_c}{2}\right)}{\frac{b_c \cdot t_c^3}{12}}$$

$$\frac{380}{4} = \frac{(48821.83)(t_c/2)}{\frac{45 \cdot t_c^3}{12}}$$

$t_c^2 = 68.52$

$t_c = 8.28\ mm$

$\boxed{t_c = 10mm}$

(v) whipping stress,

$$(M_b)_{max} = m_1 r \omega^2 \frac{L^2}{9\sqrt{3}}$$

$$m_1 = A\rho$$

$$= (11t^2)(7800)$$

$$\cancel{= [11(5.5)^2][7800]}$$

$$\cancel{= 2595480}$$

$$= 11(0.005)^2 (7800)$$

$$\boxed{m_1 = 2.145\ kg/m}$$

$$(M_b)_{max} = (2.145)(0.070)(209.4)^2 \frac{(0.350)^2}{9\sqrt{3}}$$

$$\boxed{(M_b)_{max} = 62.74\ N\text{-}m}$$

$$(M_b)_{max} = (62.74 \times 10^3)\ N\text{-}mm.$$

$$I_{xx} = \left(\frac{419}{12}\right)t^4, \quad y = \frac{5t}{2}$$

$$\sigma_b = \frac{M_b y}{I}$$

$$\boxed{\sigma_b = 27\ N/mm^2}$$ — whipping stress.

* Crankshaft

- Crankshaft → connects Reciprocating motion to Rotatory motion. through connecting rod.

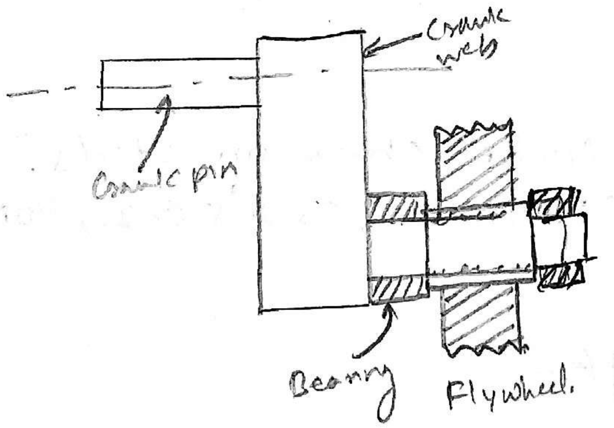
Crankshaft ~~part~~ Portions.

- Crankpin.
- Crank web
- Shaft.

- Shaft portion rotates in ~~b~~ main bearing.

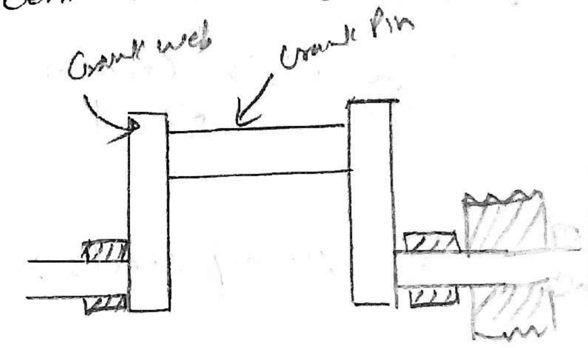
- Types of crankshaft
  - Side crankshaft
  - Centre crankshaft.

Side crankshaft




* - overhung Crankshaft.
- only one crank web.
- require only two Bearings
- used for medium size Engines

Centre crankshaft.




- Two webs
- three bearings required
- used in radial aircraft engines, stationary Engines marine engines

- Types of crankshaft also ~~cal~~ classified as
  - Single throw (one crank pin)
  - multi throw (more than one crank pin multi cylinder Engine.

- loads on Crankshaft:-

(1) strength to withstand bending & twisting moments.

(2) — " — " — " — lateral and angular deflections within permissible limits.

(3) Sufficient endurance limit to withstand fluctuating stresses.

- manufacturing of crankshaft:-

Drop forging Process.

- materials of Crankshaft:-

(1) Plain Carbon steel :- 40C8, 45C8, 50C4.

(2) Alloy steels :- Nickel chromium steels. (16Ni3Cr2, 35Ni5Cr2, 40Ni10Cr3Mo6)

- Design of Crankshaft:-

~~force exerted~~ Bending & twisting moment is due to following three forces:-

(1) Force exerted by connecting Rod on Crankpin

(2) wt. of flywheel in vertical direction.

(3) Resultant ~~belt~~ belt tensions acting in the horizontal direction

Position of cranks to be considered for design.

Case I

Crank is at T.D.C

Subjected to maximum bending moment and no torsional moment

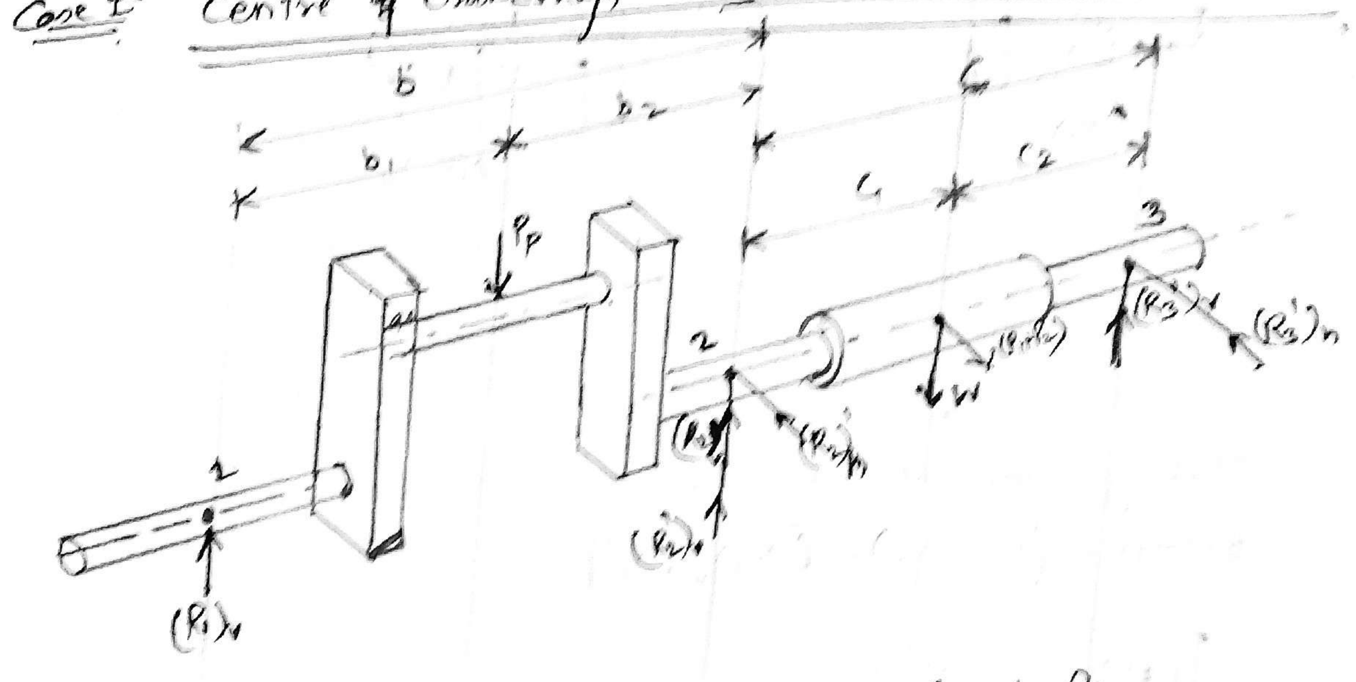
Case II

Crank is at angle with the line of dead centre positions & subjected to maxm torsional moment

$\theta = 25^\circ$ to $35^\circ$ (Petrol Engine)

$\theta = 30^\circ$ to $40^\circ$ (diesel Engine)

Case I Centre of crankshaft at T.D.C Position:-




$P_p \rightarrow$ Force acting on Crank Pin.

Assumptions

① Engine is vertical & crank is at T.D.C

② Belt drive is horizontal

③ Crankshaft is simply supported on bearings.

(A) Bearing Reactions

Reaction on bearing 1 & 2 due to $P_p$
$(R_1)_v$ and $(R_2)_v$

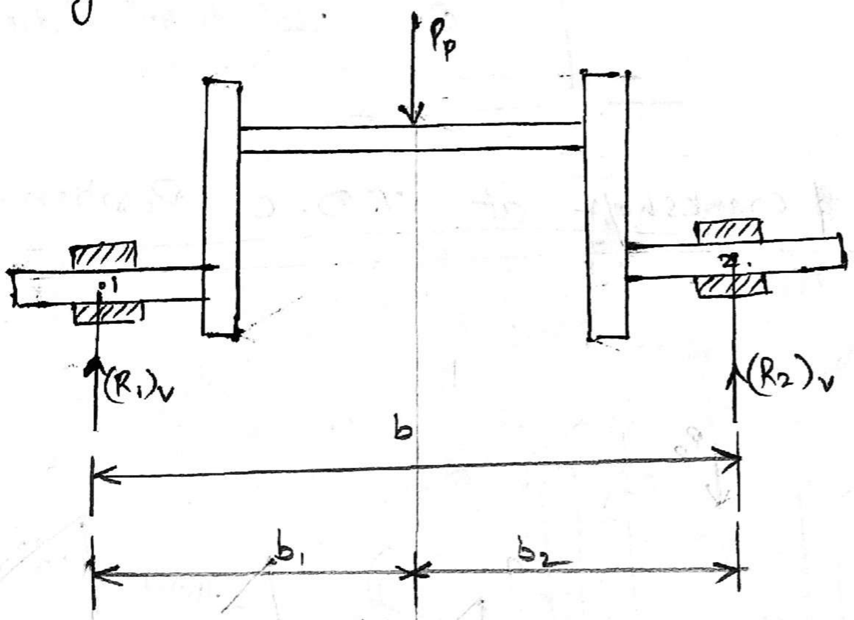
Reaction on bearing 2 & 3 due to flywheel wt. (W) and belt tension $(P_1+P_2)$
$(R_2')_v$, $(R_2')_h$ and $(R_3')_v$, $(R_3')_h$.

at T.D.C.,

$$P_c = P_p. \quad \left( \because \frac{\text{Thrust in connecting rod}}{} = \text{force acting on Piston.} \right)$$

$$\boxed{P_p = P_{max} \cdot \left(\frac{\pi}{4} \cdot D^2\right)}$$

Assuming the beam between 1 and 2 as simply supported,




Taking moment at 1.

$$\Sigma M_1 = 0 = (P_p \cdot b_1) - [(R_2)_v \cdot b]$$

$$\boxed{(R_2)_v = \frac{P_p \cdot b_1}{b}}$$

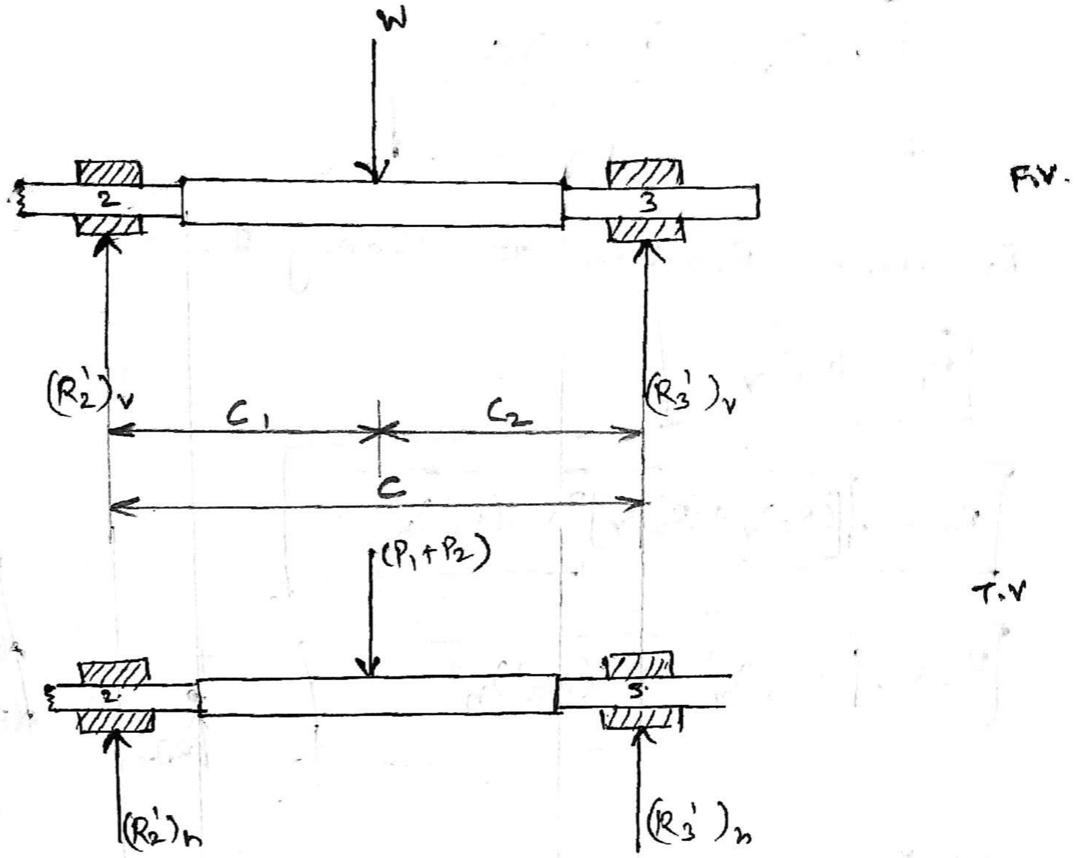
Similarly,

Taking moment at 2

$$\Sigma M_2 = 0 = -(P_p \cdot b_2) + [(R_1)_v \cdot b]$$

$$\boxed{(R_1)_v = \frac{P_p \cdot b_2}{b}}$$

Similarly assuming the shaft between 2 & 3 as simply supported beam.




Taking moment at 2 in vertical plane,

$$\Sigma M_2 = 0 = [W \cdot c_1] - [(R_3')_v \cdot c]$$

$$\boxed{(R_3')_v = \frac{W \cdot c_1}{c}}$$

Taking moment at 3 in vertical plane,

$$\Sigma M_3 = 0 = +[(R_2')_v \cdot c] \mp [W \cdot c_2]$$

$$\boxed{(R_2')_v = \frac{W c_2}{c}}$$

Similarly Taking moment at 2 in horizontal plane,

$$\Sigma M_2 = 0 = [(P_1+P_2) \cdot c_1] - [(R_3')_h \cdot c]$$

$$\boxed{(R_3')_h = \frac{(P_1+P_2)\, c_1}{c}}$$

Similarly taking moment at 3 in horizontal plane.

$$\Sigma m_3 = 0 = \left[(R_2')_h \cdot c\right] - \left[(P_1+P_2) \cdot c_2\right]$$

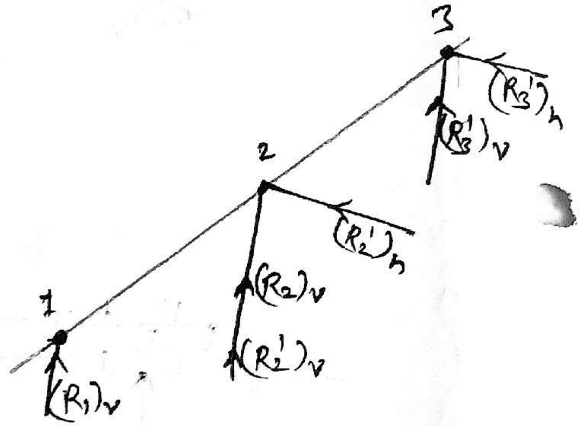
$$\boxed{(R_2')_h = \frac{(P_1+P_2)\, c_2}{c}}$$

Resultant Reaction at Bearing 1.

$$\boxed{R_1 = (R_1)_v}$$

$$\boxed{R_2 = \sqrt{\left[(R_2)_v + (R_2')_v\right]^2 + (R_2')_h^2}}$$

$$\boxed{R_3 = \sqrt{(R_3')_v^2 + (R_3')_h^2}}$$

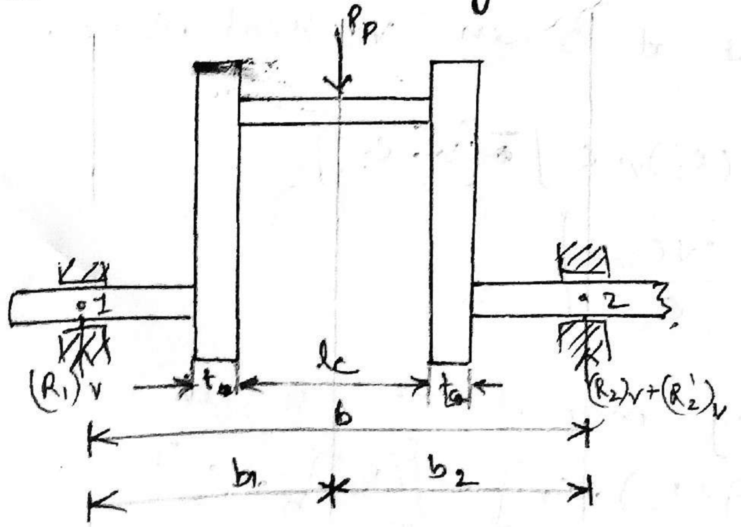



Note* :-

① if b not given then, take $\boxed{b = 2D}$

(B) Design of Crank Pin

Design will be done on the basis of ~~Crank Pin~~ bending stress consideration.




dc

$$(M_b)_c = (R_1)_v \cdot b$$

$$I = \frac{\pi}{64} d_c^4 \quad , \quad Y = \frac{d_c}{2}$$

$$\boxed{\sigma_b = \frac{(M_b)_c \cdot Y}{I}}$$

if $\sigma_b$ not given,

take it as $75\ N/mm^2$.

Similarly,

length of crank pin is determined by bearing consideration.

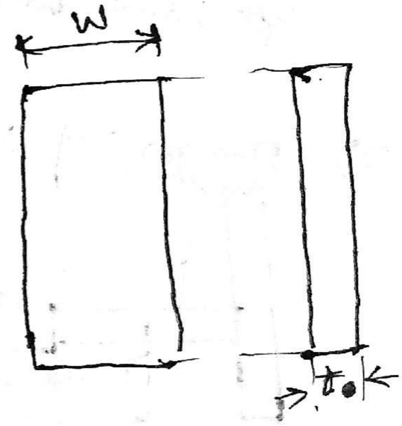
$P_b$ = allowable bearing pressure at crank pin bush.

$$\boxed{P_b = \frac{P_p}{l_c \cdot d_c}}$$

(c) Design of left-hand crank web.

By Empirical relationship,

$$\boxed{\begin{aligned} t &= 0.7\, d_c \\ w &= 1.14\, d_c \end{aligned}}$$




This dimensions can be checked for stresses by,

In web there are two stresses due to Reactions,

① direct compressive

② Bending stress.

① direct compressive

$$\sigma_c = \frac{(R_1)_v}{w t}.$$

(2) Bending stresses:-

Bending will occur at central plane of web due to reactions. eccentricity.

$$(M_b) = (R_1)_v \left[ b_1 - \frac{l_c}{2} - \frac{t}{2} \right]$$

$$I = \frac{1}{12}(w)(t^3) \quad , \quad Y = \frac{t}{2}.$$

$$\sigma_b = \frac{M_b \cdot Y}{I}.$$

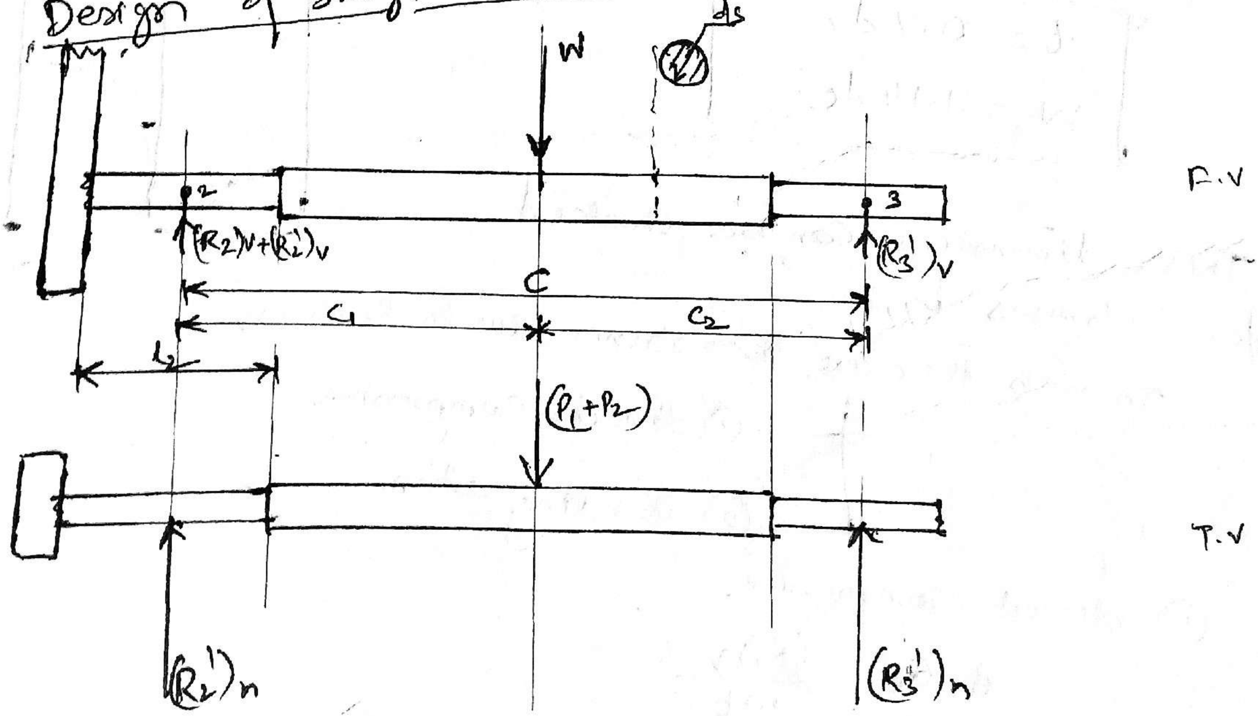
$$\sigma_b = \frac{(R_1)_v \left[ b_1 - \frac{l_c}{2} - \frac{t}{2} \right] \cdot \frac{t}{2}}{\frac{w t^3}{12}}$$

$$\boxed{\sigma_b = \frac{6 (R_1)_v \left[ b_1 - \frac{l_c}{2} - \frac{t}{2} \right]}{w t^2}}$$

(C) Design of Right Hand web.

The Right hand web should be identical to left hand web. from balancing consideration.

(D) Design of Shaft Under Flywheel:-



[illegible] 1 & 2 (crank pin) [illegible]

Flywheel.

$d_s$ = dia. of shaft under flywheel.

(1) ~~[illegible]~~. B.M in vertical plane.

$$(M_b)_v = (R_3')_v \cdot C_2$$

(2) B.M. in horizontal plane.

$$(M_b)_h = (R_3')_h \cdot C_2$$

(3) Resultant B.M. on shaft.

$$M_b = \sqrt{(M_b)_v^2 + (M_b)_h^2}$$

$$= \sqrt{[(R_3')_v \cdot C_2]^2 + [(R_3')_h \cdot C_2]^2}$$

$$\boxed{M_b = C_2 \sqrt{(R_3')_v^2 + (R_3')_h^2}}$$

$$I = \frac{\pi}{64} d_s^4 , \quad y = \frac{d_s}{2} , \quad \sigma_b = \frac{M_b y}{I}.$$

Case II

## Centre crankshaft at angle of maximum torque :-

A) ~~Bearing Re~~ Components of load when the tangential

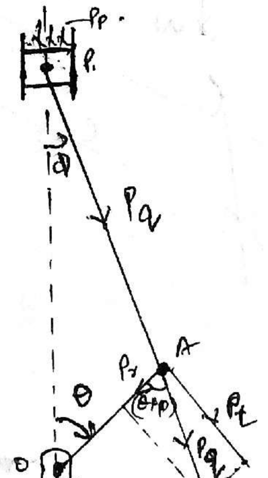
The torque is maximum

Component of force on crank pin is maximum.

i.e. at angle

$\theta = 25^\circ$ to $35^\circ$ (petrol Engines)

$\theta = 30^\circ$ to $40^\circ$ (diesel Engines)



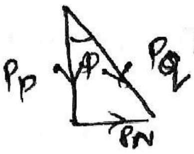
let $p'$ = Pressure acting on piston during maximum torque condition.

$$P_p = \left[\frac{\pi}{4} D^2\right] p'$$

we know $\quad \sin\phi = \dfrac{\sin\theta}{n}$

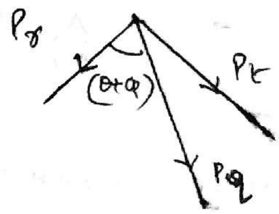
So,

$$\cos\phi = \frac{P_p}{P_q}$$

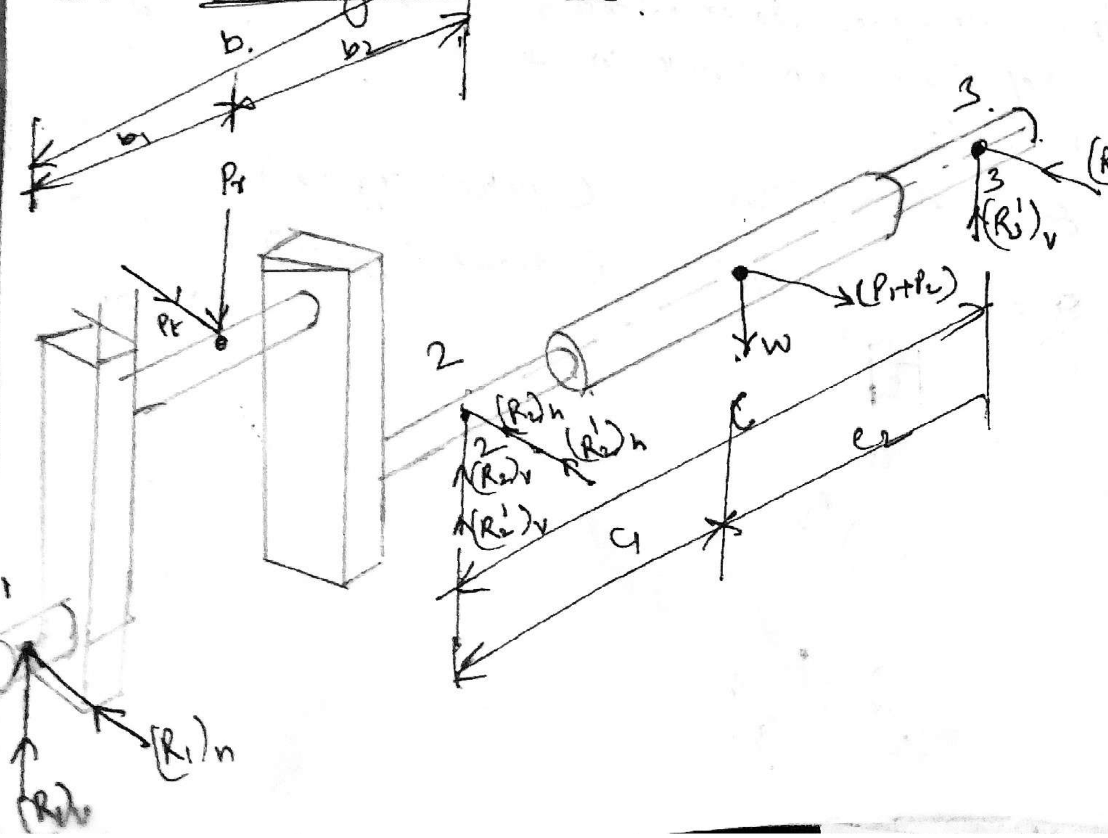



$$\boxed{P_q = \frac{P_p}{\cos\phi}}$$

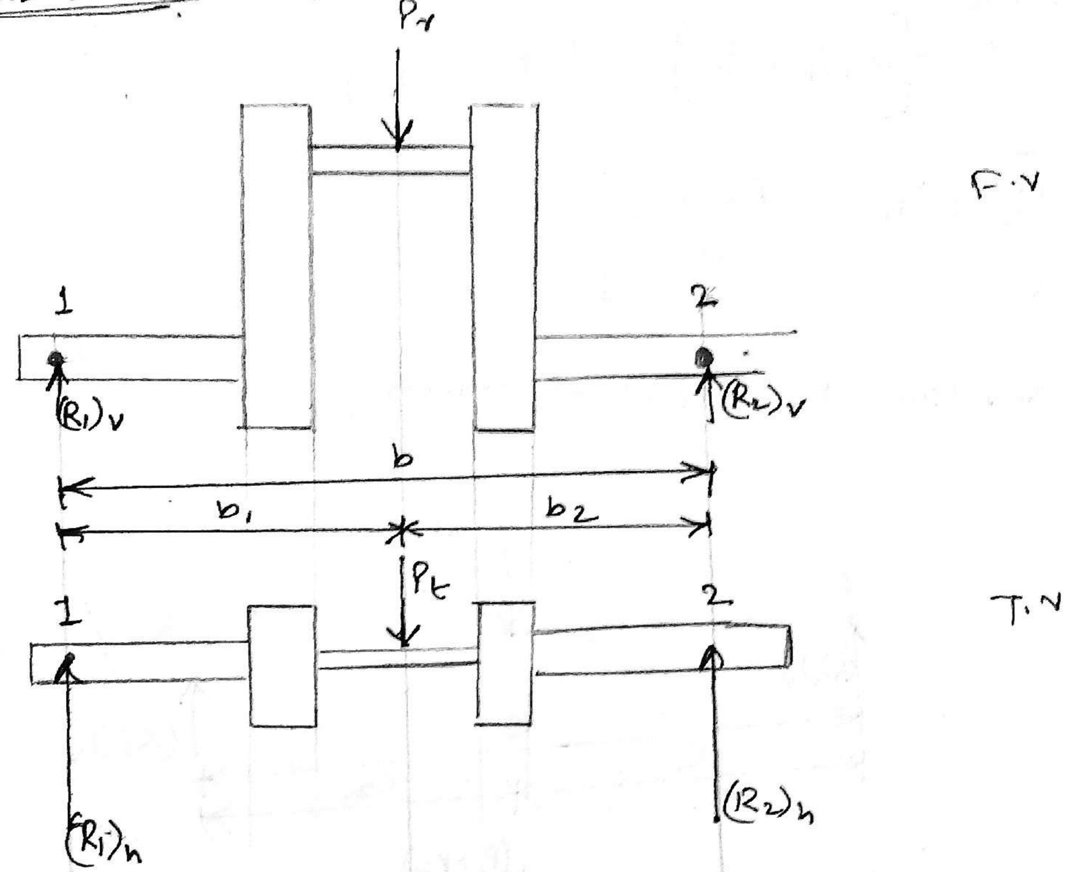
Similarly
$$\boxed{\begin{aligned} P_t &= P_q \sin(\theta+\phi) \\ P_r &= P_q \cos(\theta+\phi) \end{aligned}}$$




(B) Bearing Reactions



① Reactions at 1 & 2 (Crank Pin) Position.




In ~~horizo~~ vertical plane,

$$\Sigma M_1 = 0$$
$$0 = (P_r \cdot b_1) - [(R_2)_v \cdot b]$$

$$\boxed{(R_2)_v = \frac{P_r \cdot b_1}{b}}$$

$$\Sigma M_2 = 0$$
$$0 = [(R_1)_v \cdot b] - [P_r \cdot b_2]$$

$$\boxed{(R_1)_v = \frac{P_r \cdot b_2}{b}}$$

Similarly in horizontal plane,

$$\Sigma M_1 = 0 = [P_t \cdot b_1] - [(R_2)_h \cdot b]$$

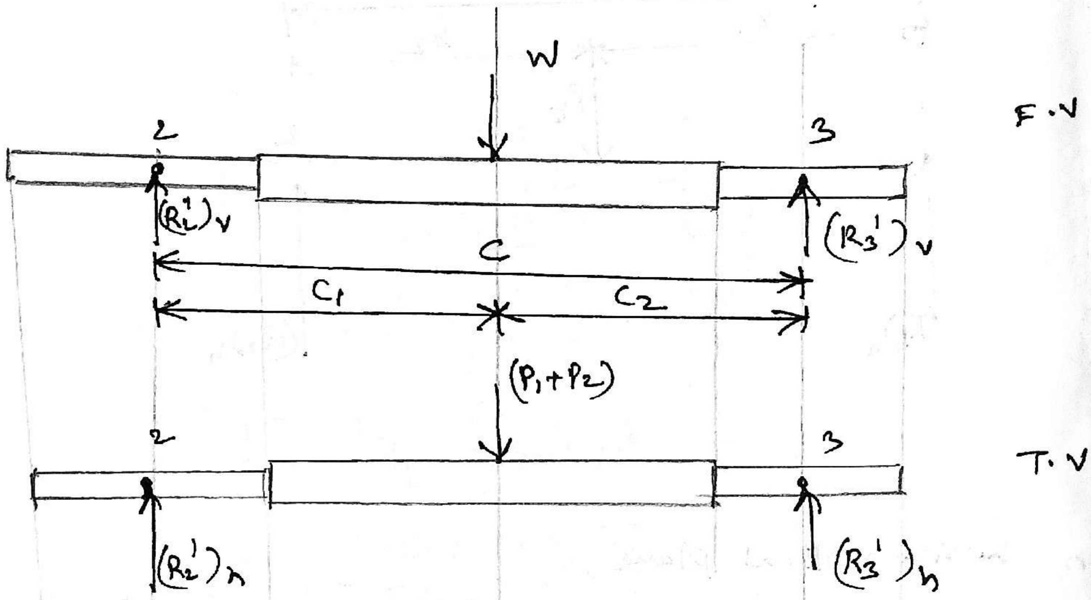
$$\boxed{(R_2)_h = \frac{P_t \cdot b_1}{b}}$$

$\Sigma M_2 = 0$

$$0 = \left[(R_1)_h \cdot b\right] - \left[P_t \cdot b_2\right]$$

$$\boxed{(R_1)_h = \frac{P_t \cdot b_2}{b}}$$

(2) Reaction at Flywheel Portion




Reaction in vertical plane:

$$\Sigma M_2 = 0 = -\left[(R_3')_v \cdot c\right] + \left[W \cdot c_1\right]$$

$$\boxed{(R_3')_v = \frac{W \cdot c_1}{c}}$$

$$\Sigma M_3 = 0 = \left[(R_2')_v \cdot c\right] - \left[W \cdot c_2\right]$$

$$\boxed{(R_2')_v = \frac{W \cdot c_2}{c}}$$

Reactions in horizontal plane:-

$$\Sigma M_2 = 0 = \left[(P_1+P_2) \cdot c_1\right] - \left[(R_3')_h \cdot c\right]$$

$$\boxed{(R_3')_h = \frac{(P_1+P_2)\,c_1}{c}}$$

$\Sigma M_3 = 0$

$$0 = -\left[(P_1+P_2) \cdot C_2\right] + \left[(R_2')_h \cdot c\right]$$

$$\boxed{(R_2')_h = \frac{(P_1+P_2)\, C_2}{c}}$$

Resultant Reactions :-

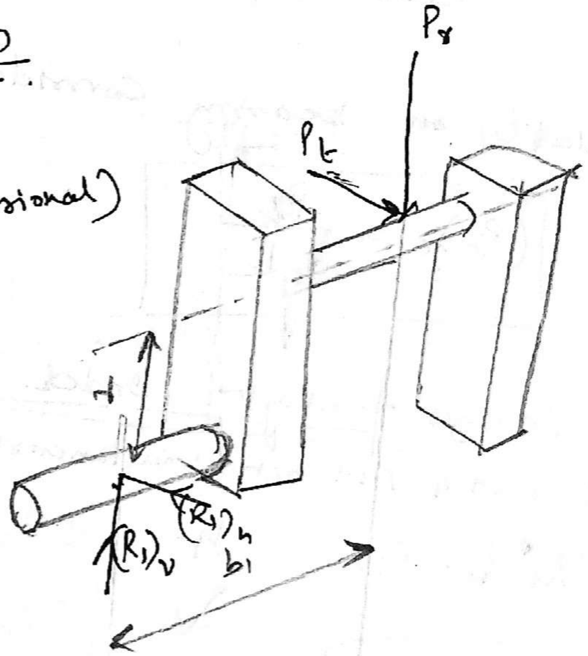
$$R_1 = \sqrt{(R_1)_h^2 + (R_1)_v^2}$$

$$R_2 = \sqrt{\left[(R_2')_v + (R_2)_v\right]^2 + \left[(R_2)_h + (R_2')_h\right]^2}$$

$$R_3 = \sqrt{\left[(R_3')_v^2 + (R_3)_v^2\right]}$$

(C) Design of crank pin

In crank pin both. Bearing & (Bending + torsional) stress is being acting.




$\cancel{(M_b)_c = (R_1)_h \cdot x}$

Bending moment,

$$(M_b)_c = (R_1)_v \cdot b_1$$

Torsion moment,

$$(M_t)_c = (R_1)_h \cdot r.$$

Since Both torsional and B.M are acting together so we will use maxm Shear stress theory.

$$\tau_{max} = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau)^2}$$

$$\tau_{max} = \sqrt{\left(\frac{M_b y}{2I}\right)^2 + \left(\frac{M_t y}{J}\right)^2}$$

$$\tau_{max} = \sqrt{\left(\frac{M_b \cdot \left(\frac{d_c}{2}\right)}{\frac{\pi}{64} d_c^4 \times 2}\right)^2 + \left(\frac{M_t \cdot \frac{d_c}{2}}{\frac{\pi}{32} \times d_c^4}\right)^2}$$

$$\tau_{max} = \sqrt{\left(\frac{16 M_b}{\pi d_c^3}\right)^2 + \left(\frac{16 M_t}{\pi d_c^3}\right)^2}$$

$$\tau_{max} = \frac{16}{\pi d_c^3}\left[\sqrt{M_b^2 + M_t^2}\right]$$

$$\boxed{d_c^3 = \frac{16}{\pi \tau_{max}}\left[\sqrt{\left[(R_1)_v \cdot b_1\right]^2 + \left[(R_1)_h \cdot r\right]^2}\right]}$$

If allowable shear stress ($\tau_{max}$) is not given

then, $\tau_{max} = 40\ N/mm^2$

Similarly on bearing considerations.

$$\boxed{(P_b)_c = \frac{P}{d_c l_c}}$$

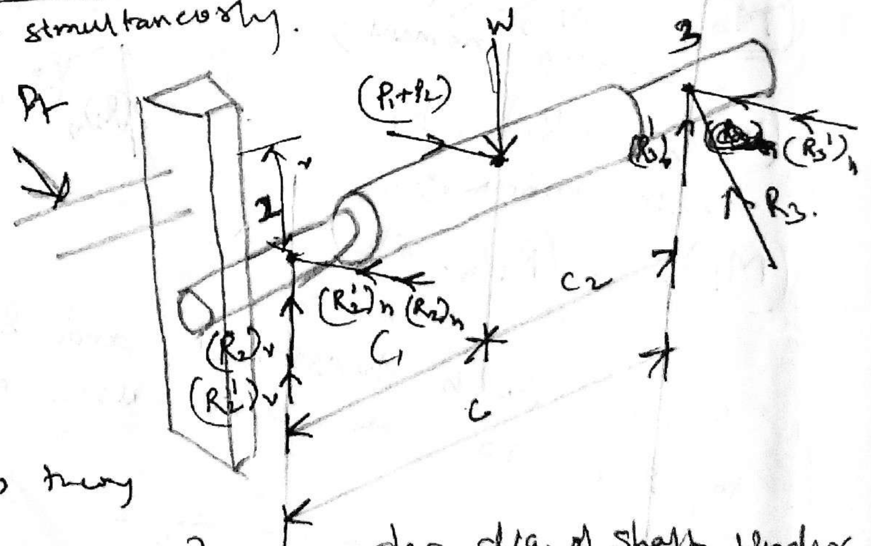
(D) Design of shaft under Flywheel:-

Both B.M & T.M act simultaneously.

$$(M_b)_s = (R_3)_v \cdot C_2$$

$$\cancel{(M_t)_s = (R_3')_h \times d_1}$$

$$(M_t)_s = (P_t) \cdot r$$




So Max$^m$ shear stress theory

$$\tau_{max} = \frac{16}{\pi d_s^3}\left[\sqrt{(M_b)_s^2 + (M_t)_s^2}\right]$$

$d_s$ = dia. of shaft under flywheel.

$$\boxed{d_s^3 = \frac{16}{\pi \tau_{max}}\left[\sqrt{\left[(R_3)_v \cdot C_2\right]^2 + \left[P_t \cdot r\right]^2}\right]}$$

(E) Design of Shaft at juncture of Right hand crank web

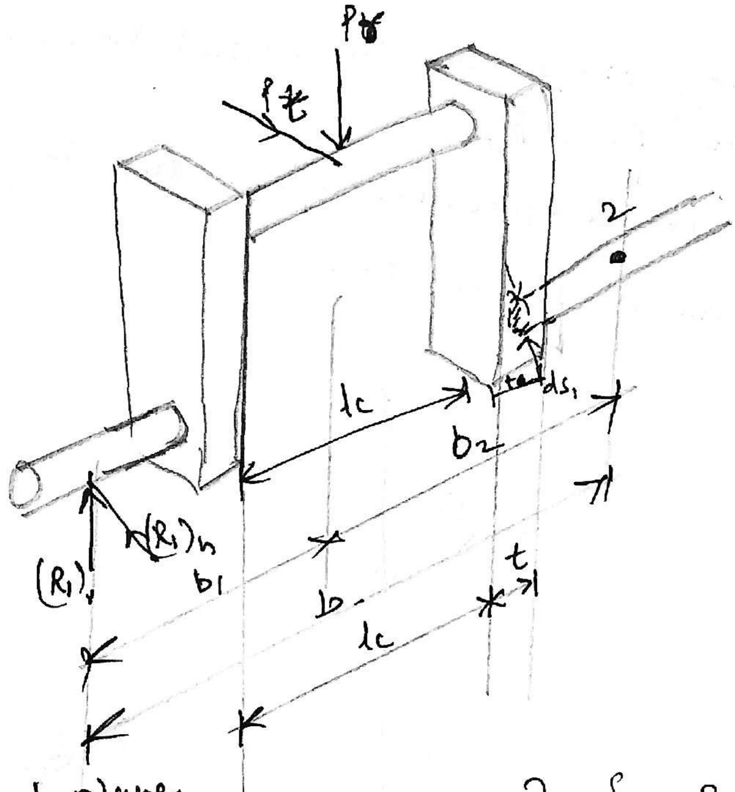
$d_{s1}$ = diameter of shaft at juncture of right hand crank web.

Moments acting at juncture:-

① B.M in vertical plane due to $(R_1)_v$ and $P_r$

② B.M in horizontal plane due to $(R_1)_h$ and $P_t$

③ T.M due to tangential component $P_t$.




B.M in vertical plane.

$$(M_b)_v = \left\{(R_1)_v \cdot \left[b_1 + \frac{l_c}{2} + \frac{t}{2}\right]\right\} - \left\{P_r \cdot \left[\frac{l_c}{2} + \frac{t}{2}\right]\right\}$$

B.M in ~~horizont~~ horizonal plane

$$(M_b)_h = \left\{(R_1)_h \cdot \left[b_1 + \frac{l_c}{2} + \frac{t}{2}\right]\right\} - \left\{P_t \left[\frac{l_c}{2} + \frac{t}{2}\right]\right\}$$

Resultant B.M

$$\boxed{M_b = \sqrt{(M_b)_v^2 + (M_b)_h^2}}$$

$$\boxed{M_t = P_t \cdot r}$$

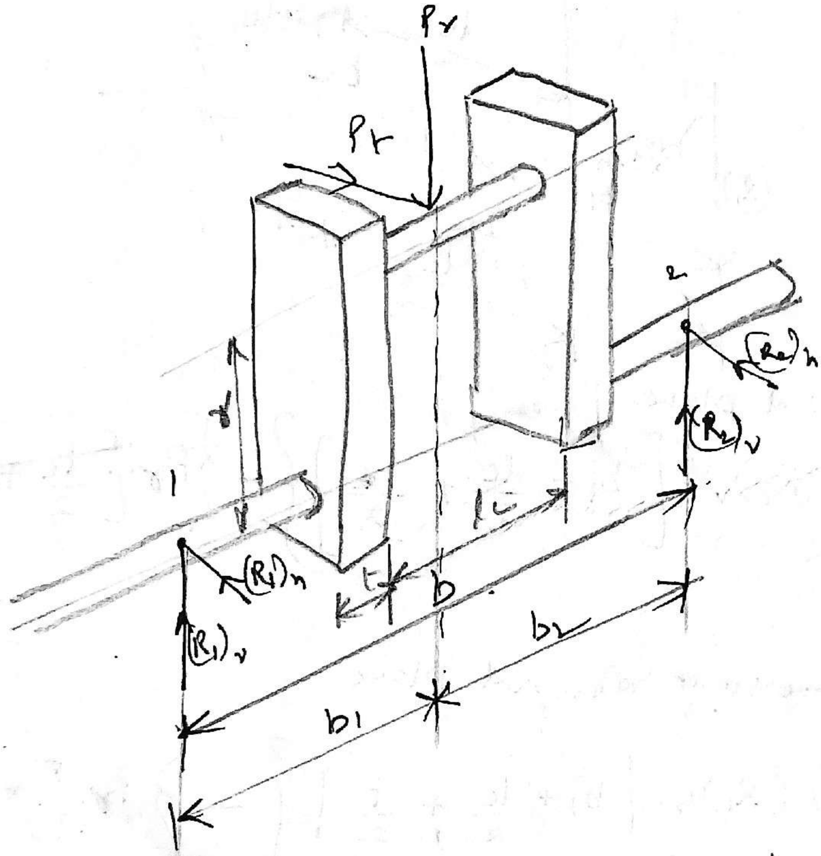
Using Max$^m$ shear stress Theory,

$$\boxed{d_{s_1}^3 = \frac{16}{\pi \tau_{max}} \sqrt{(M_b)^2 + (M_t)^2}}$$

## (F) Design of Right hand crank web:-

Right hand crank web is subjected to following stresses:-

1. Bending stress due in vertical horizontal comp. due to $P_r$ & $P_t$
2. Direct compressive stress due to $P_r$
3. Torsional shear stress




Bending moment due to radial component

$$(M_b)_r = (R_2)_v \left[ b_2 - \frac{l_c}{2} - \frac{t}{2} \right]$$

$$(\sigma_b)_r = \frac{(M_b)_r \cdot \left(\frac{t}{2}\right)}{\frac{1}{12}(w)\,t^3}$$

$$\boxed{(\sigma_b)_r = \frac{6\,(M_b)_r}{w t^2}}$$

Similarly ~~in~~ ~~do~~ B.M due to $P_t$

$$(M_b)_t = P_t \cdot \left[ r - \frac{ds_1}{2} \right]$$

$$(\sigma_b)_t = \frac{(M_b)_t \cdot y}{I}$$

$$= \frac{(M_b)_t \cdot \left(\frac{w}{2}\right)}{\frac{1}{12}(t)\,w^3}$$

$$\boxed{(\sigma_b)_t = \frac{6\,(M_b)_t}{t w^2}.}$$

Similarly the direct compressive stress,

$$\boxed{(\sigma_c)_d = \frac{P_r}{2wt}}$$

Maximum compressive stress is given by

$$\boxed{\sigma_c = (\sigma_b)_r + (\sigma_b)_t + (\sigma_c)_d}$$

Similarly torsional moment on arm is given by.

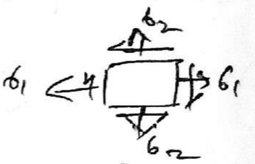
$$M_t = (R_1)_h \left[ b_1 + \frac{l_c}{2} \right] - P_t \left[ \frac{l_c}{2} \right]$$

or

$$M_t = (R_2)_h \left[ b_2 - \frac{l_c}{2} \right]$$

$$\tau = \frac{M_t \cdot r}{J}$$

$$= \frac{\cancel{M_t \left(\frac{t}{2}\right)}}{\cancel{\frac{wt^3}{9}}} \qquad \boxed{\tau = \frac{4.5\, M_t}{w t^2}}$$




$\sigma_{max} = \left(\frac{\sigma_1 - \sigma_2}{2}\right) \pm \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2 + \tau^2}$

maximum compressive stress is given by

$$\boxed{(\sigma_c)_{max} = \frac{\sigma_c}{2} + \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\tau^2}}$$

## (G) Design of Left crank web:-

Same as Right hand i.e identical to Right hand web.

## (H) Design of Crankshaft Bearing:-

Bearing 2 is subjected to maximum stress, i.e

$$R_2 = \sqrt{\left[(R_2)_v + (R_2')_v\right]^2 + \left[(R_2)_h + (R_2')_h\right]^2}$$

So design of bearing is done on Bearing considered

$$\boxed{P_b = \frac{R_2}{d_{s1}\, l_2}}$$

25.18

$D = 125$ mm

$n = 4.5$

$P_{max} = 2.5$ N/mm²

$L = 150$ mm

$r = \frac{L}{2} = 75$ mm

$W = 1$ KN $= 10^3$ N.

$P_1 + P_2 = 2 \times 10^3$ N

Width of hub = 200mm.

Torque is max when $\theta = 25^\circ$.

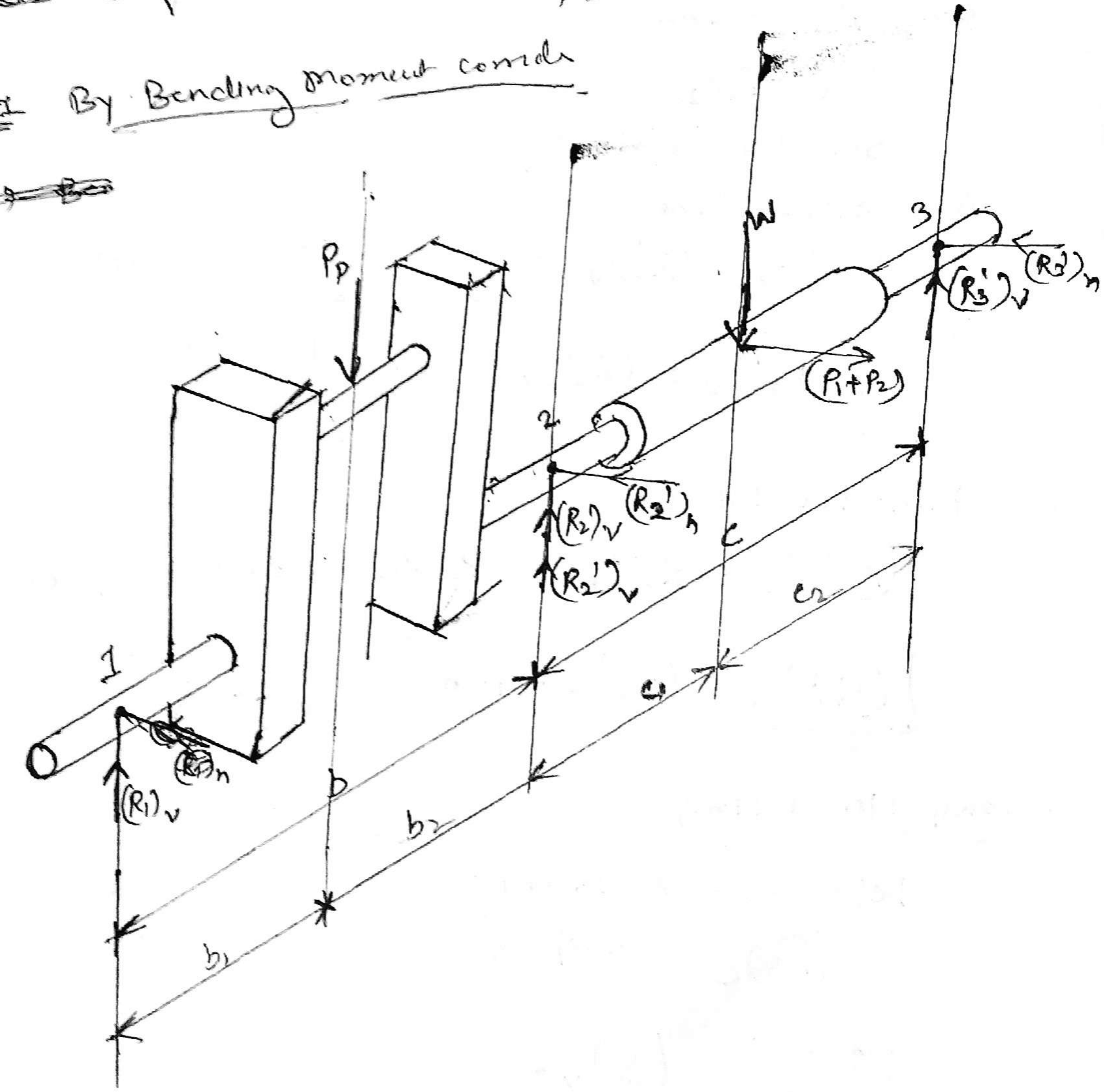
$P = 2$ MPa

$\frac{l_c}{d_c} = 1$

allowable $\sigma_t$ for crank shaft mtrl $= 70$ N/mm²

$(P_b)$ crank pin $= 10$ N/mm²

Case I  By Bending moment condn




$P_p = (P_{max})\left(\frac{\pi}{4}D^2\right)$

$= (2.5)\left(\frac{\pi}{4} \times 125^2\right)$

$\boxed{P_p = 30679.62 N}$

$b = 2D$ — by empirical relationship.

$= 2(125)$

$\boxed{b = 150 \text{ mm}}$

(A) Reactions

let $b_1 = b_2 = \frac{150}{2} = 125 \text{ mm}$.

By symmetry.

$(R_1)_v = (R_2)_v = \left(\frac{P_p}{2}\right) = \frac{30679.62}{2}$

$\boxed{(R_1)_v = (R_2)_v = 15339.81 \text{N}}$

similarly assume.

$c_1 = c_2$

so, by symmetry

in vertical plane.

$(R_2')_v = (R_3')_v = \frac{W}{2} = \frac{10^3}{2} = 500 \text{N}$

$\boxed{(R_2')_v = (R_3')_v = 500 \text{N}}$

In horizontal plane,

$(R_2')_h = (R_3')_h = \frac{P_1 + P_2}{2} = \frac{2 \times 10^3}{2} = 1000 \text{N}$

$\boxed{(R_2')_h = (R_3')_h = 1000 \text{N}}$

(B) Crank Pin design

let $\sigma_b = 75 \text{ N/mm}^2$.

$(P_b)_c = 10 \text{N/mm}^2$

$(M_b)_c = (R_1)_v \cdot b_1$

$= [15339.81 \times 125]$

$\boxed{(M_b)_c = 1917.48 \times 10^3 \text{ N-mm}}$

$$y = \frac{d_c}{2} \Rightarrow I = \frac{\pi}{64} d_c^4$$

$$\sigma_b = \frac{(M_b)_c \left(\frac{d_c}{2}\right)}{\frac{\pi}{64} d_c^4}$$

$$75 = \frac{(1917.48 \times 10^3)\left(\frac{d_c}{2}\right)}{\frac{\pi}{64} d_c^{4}}$$

$$75 = \frac{32 \times 1917.48 \times 10^3}{\pi d_c^3}$$

$$d_c = 63.86 \text{ mm}$$

or

$$\boxed{d_c = 65 \text{ mm}}$$

let us assume that $\frac{l_c}{d_c} = 1$

$$(P_b)_c = \frac{P_p}{d_c l_c}$$

$$(P_b)_c = \frac{30679.62 N}{(65)(65)}$$

$$\boxed{(P_b)_c = 7.26 \text{ N/mm}^2}$$

$(P_b)_c < 10 \text{ N/mm}^2$

So it is

$$\boxed{l_c = d_c = 65 \text{mm}}$$

(c) Design of left hand crank web:-

By empirical relationships,

$$t = 0.7 d_c$$
$$= 0.7(65)$$
$$\boxed{t = 45.5 \text{ mm}} \quad \text{or} \quad \boxed{t = 46 \text{ mm}}$$

$$w = 1.14 d_c$$
$$= 1.14(65)$$
$$\boxed{w = 74.1\,mm} \quad \text{or} \quad \boxed{w = 75\,mm}$$

There are (1) Direct compressive stress
(2) Bending stress in webs.

$$\sigma_c = \frac{(R_1)_v}{wt} = \frac{15339.81}{(75)(46)} = 4.45\ N/mm^2$$

$$\boxed{\sigma_c = 4.45\ N/mm^2}$$

$$\sigma_b = \frac{(R_1)_v\left[b_1 - \frac{l_c}{2} - \frac{t}{2}\right]\left[\frac{t}{2}\right]}{\frac{1}{12}[wt^3]}$$

$$= \frac{15339.81 \times 6\left[125 - \frac{65}{2} - \frac{46}{2}\right]}{[75][46]^2}$$

$$\boxed{\sigma_b = 40.31\ N/mm^2}$$

$$(\sigma_c)_t = \sigma_c + \sigma_b$$
$$= 4.45 + 40.31$$
$$\boxed{(\sigma_c)_t = 44.76\ N/mm^2}$$

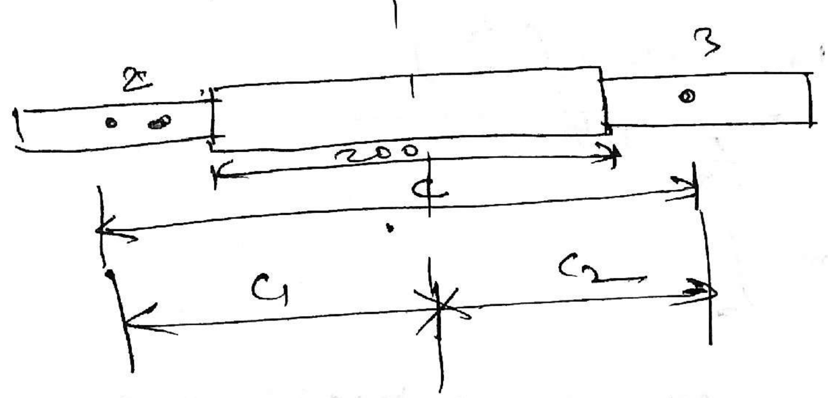
Total compressive load $(\sigma_c)_t < 70\ N/mm^2$
Design is Safe.

(D) Design of Right hand crank web:-

It is identical to left hand crank web.

(E) Design of Shaft under flywheel;

width of hub = 200mm.



$C = 200 +$ margin for length of two bearings

$= 200 + 100$

$C = 300$

$C_1 = C_2 = \frac{300}{2} = 150\,mm$

$\boxed{C_1 = C_2 = 150\,mm}$

$(M_b)_v = (R_3')_v \cdot C_2$

$\boxed{(M_b)_v = (500)(150) = 75 \times 10^3\ N\text{-}mm}$

$(M_b)_h = (R_3')_h \cdot C_2$

$(M_b)_h = (1000)(150)$

$\boxed{(M_b)_h = 150000\ N\text{-}mm}$

$M_b = \sqrt{(M_b)_v^2 + (M_b)_h^2}$

$= \sqrt{(75 \times 10^3)^2 + (150 \times 10^3)^2}$

$\boxed{M_b = 167.71 \times 10^3\ N\text{-}mm}$

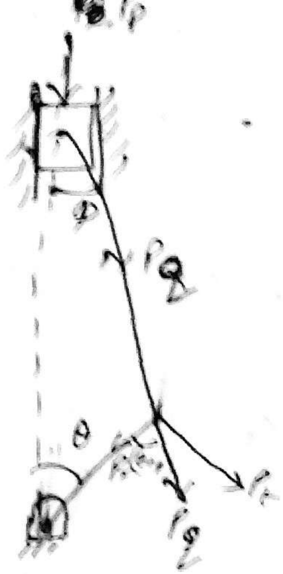
$$\sigma_b = \frac{M_b y}{I}$$

| Taking, $\sigma_b = 75\ N/mm^2$

$$75 = \frac{(167.71\times10^3)\left(\frac{d_s}{2}\right)}{\frac{\pi}{64} d_s^4}$$

$$\boxed{d_s = 30mm}$$

Case II — By turning Moment or Maximum torque considerations




It is given for $\theta = 25°$

and pressure $p' = 2\ N/mm^2$

$$P_p = \left(\frac{\pi}{4}D^2\right)p'$$

$$= \frac{\pi}{4}(125)^2\cdot(2)$$

$$\boxed{P_p = 24543.69\ N}$$




$$\cos\phi = \frac{P_p}{P_q}$$

$$P_q = \frac{P_p}{\cos\phi}$$

$$\therefore \sin\phi = \frac{\sin\theta}{n}$$

$$\sin\phi = \frac{\sin(25)}{4.5}$$

$$\boxed{\phi = 5.39°}$$

then $$P_q = \frac{P_p}{\cos\phi}$$

$$= \frac{24543.69}{\cos(5.39)}$$

$$\boxed{P_q = 24652.69\ N}$$

$$P_t = P_q \sin(\theta + \phi)$$

$$= (24652.69) \sin(25 + 5.39^\circ)$$

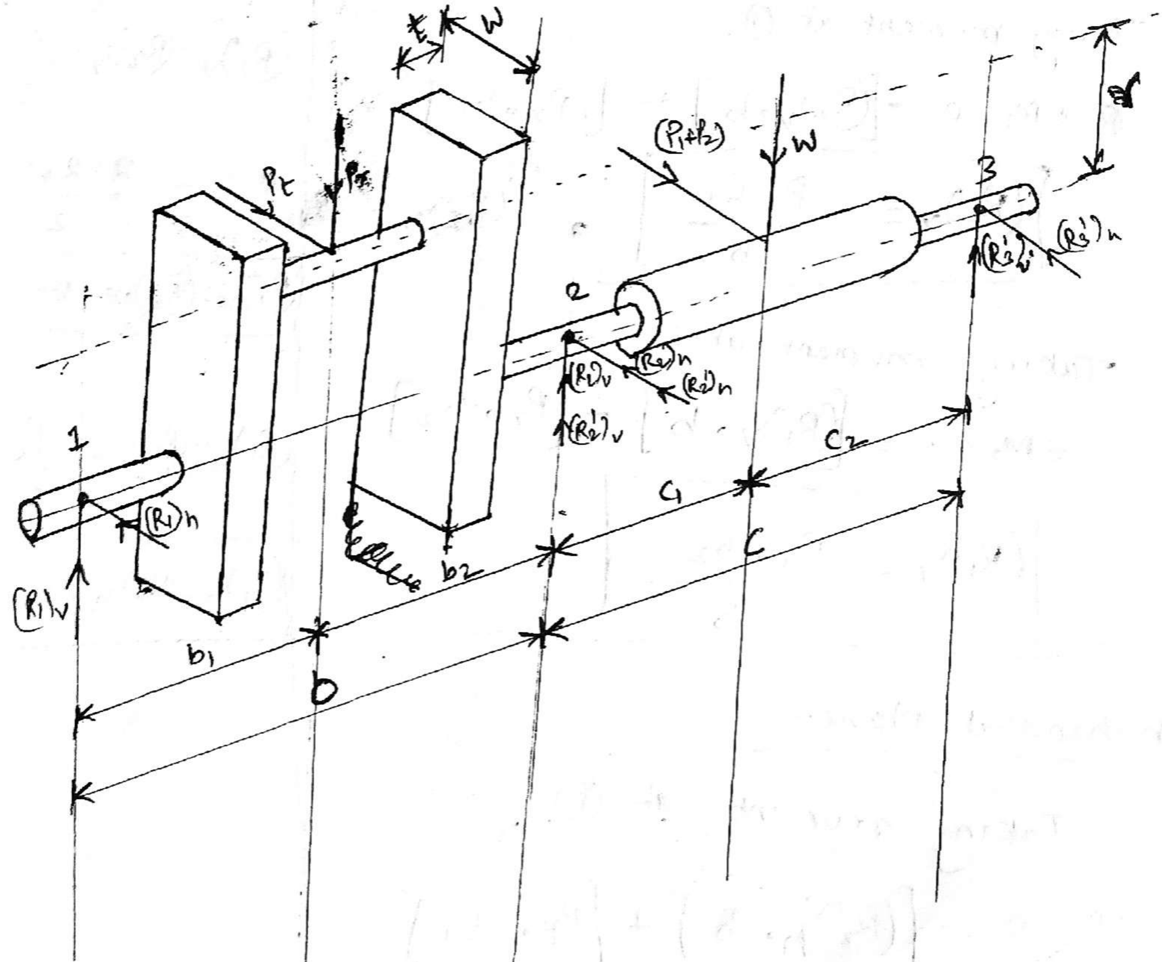
$$\boxed{P_t = 12471.38\ N}$$

$$P_r = P_q \cos(\theta + \phi)$$

$$= (24652.69) \cos(25 + 5.39)$$

$$\boxed{P_r = 21265.46\ N}$$

(A) Bearing Reactions

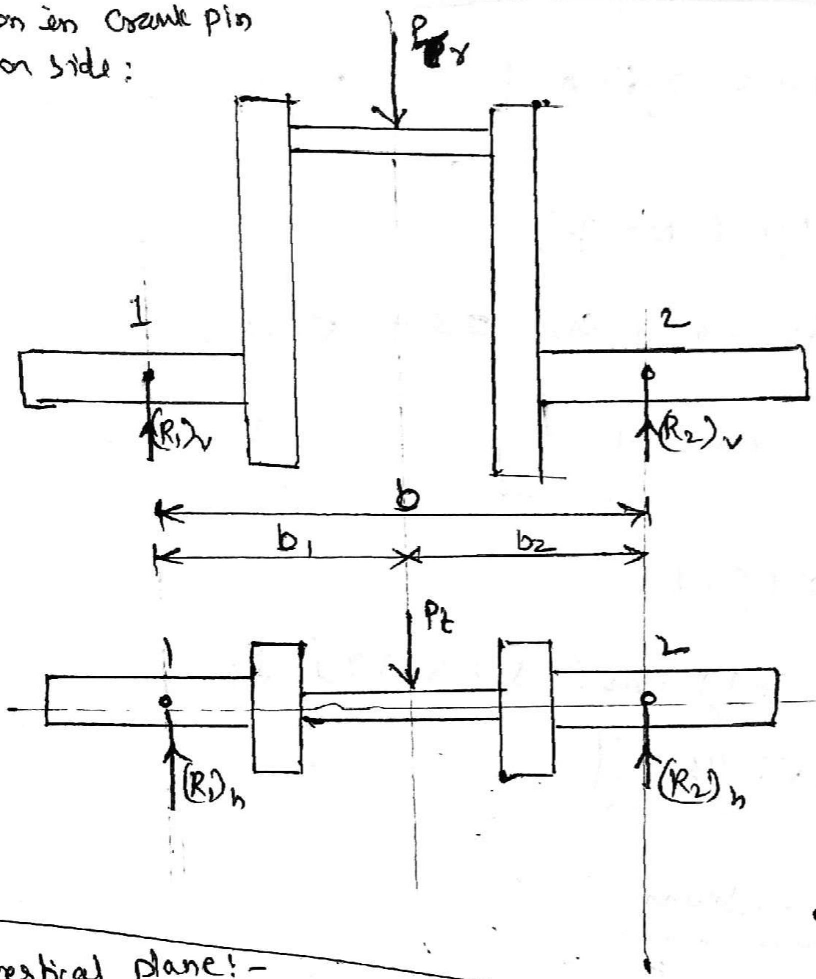



$\therefore b = 250$ , $C = 300\,mm$

$b_1 = b_2 = \frac{250}{2}$ $\quad$ $C_1 = C_2 = \frac{300}{2}$

$\boxed{b_1 = b_2 = 125\,mm}$ $\quad$ $\boxed{C_1 = C_2 = 150\,mm}$

Reaction in crank pin position side:




Problem

In vertical plane:-

Taking moment at ①.

$$\Sigma M_1 = 0 = -[(R_2)_v \cdot b] + [P_r \cdot b_1]$$

$$\boxed{(R_2)_v = \frac{P_r \cdot b_1}{b}}$$

Taking moment at 2.

$$\Sigma M_2 = 0 = [(R_1)_v \cdot b] - [P_r \cdot b_2]$$

$$\boxed{(R_1)_v = \frac{P_r \cdot b_2}{b}}$$

In horizontal plane:-

Taking moment at ①.

$$\Sigma M_1 = 0 = -[(R_2)_h \cdot b] + [P_t \cdot b_1]$$

$$\boxed{(R_2)_h = \frac{P_t \cdot b_1}{b}}$$

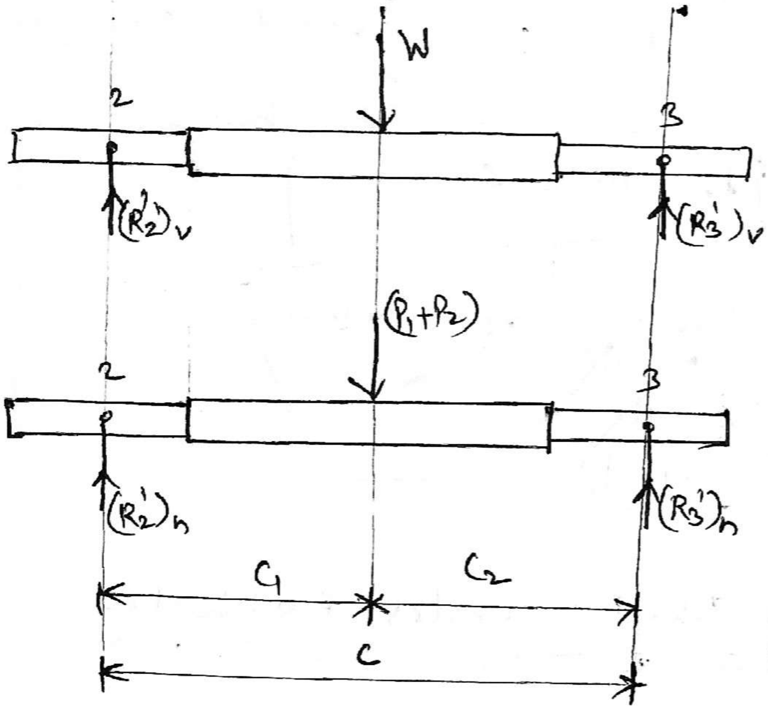
$$(R_1)_v = (R_2)_v = \frac{P_r}{2}$$

$$= \frac{21265.46}{2}$$

$$\boxed{(R_1)_v = (R_2)_v = 10632.7\,N}$$

$$(R_1)_h = (R_2)_h = \frac{P_t}{2}$$

$$\boxed{(R_1)_h = (R_2)_h = 6235.89}$$




$$(R_2')_v = (R_3')_v = \frac{W}{2} = \frac{1000}{2}$$

$$\boxed{(R_2')_v = (R_3')_v = 500\,N}$$

$$(R_2')_h = (R_3')_h = \frac{P_1+P_2}{2} = \frac{2000}{2}$$

$$\boxed{(R_2')_h = (R_3')_h = 1000\,N.}$$

(B) Design of crank Pin:-

Assume $\tau_{max} = 40\,N/mm^2$

By maximum shear stress theory,

$$d_c^3 = \frac{16}{\pi \tau_{max}} \sqrt{(M_b)^2 + (M_t)^2}.$$

$$= \frac{16}{\pi \tau_{max}} \sqrt{[(R_1)_v \cdot b_1]^2 + [(R_1)_h \cdot r]^2}$$

$$= \frac{16}{\pi (40)} \sqrt{[(10632.7)\cdot(125)]^2 + [(6235.69)(75)]^2}$$

$$\boxed{d_c = 56.4\,mm}$$

let assume,

$$\frac{l_c}{d_c} = 1 \qquad \boxed{\therefore l_c = d_c = 56.4\ mm}$$

but in previous case $d_c = 65$ hence false.

$$\boxed{l_c = d_c = 65\ mm}$$

~~Check for bearing consideration.~~

$$\cancel{(P_b)_c = \frac{P_P}{l_c d_c}}$$

(c) <u>Design of Shaft under Flywheel:-</u>

$d_s$ = dia. of Shaft under flywheel.

$$M_b = R_3 \cdot C_2$$

$$= \left[\sqrt{(R_3')_v^2 + (R_3')_h^2}\right] C_2$$

$$= \left[\sqrt{(500)^2 + (1000)^2}\right] (150)$$

$$\boxed{M_b = 1118.03\ N\text{-}mm}$$

$$M_t = P_t \cdot r$$

$$= (12471.38)(75).$$

$$\boxed{M_t = 935353.5\ N\text{-}mm}$$

By max$^m$ Shear Stress theory.

$$d_s^3 = \frac{16}{\pi \tau_{max}} \sqrt{(M_b)^2 + (M_t)^2}$$

$$= \frac{16}{\pi (40)} \sqrt{(1118.03)^2 + (935353.5)^2}$$

$d_s = 49.46\,mm \simeq 50\,mm.$

$\boxed{d_s = 50\,mm}$

(D) **Design of shaft dia. at juncture of Right hand crank web :-**

$d_{s_1}$ = diameter of shaft at juncture of Right hand crank web.

Bending moment in both vertical & horizontal plane is going to act,

$$(M_b)_v = (R_1)_v \left[ b_1 + \frac{l_c}{2} + \frac{t}{2} \right] - (P_r)\left[ \frac{l_c}{2} + \frac{t}{2} \right]$$

$$= (10632.7)\left[ 125 + \frac{65}{2} + \frac{46}{2} \right] - (21265)\left[ \frac{65}{2} + \frac{46}{2} \right]$$

$$\boxed{(M_b)_v = 738.97 \times 10^3\ N\text{-}mm}$$

$$(M_b)_h = (R_1)_h \left[ b_1 + \frac{l_c}{2} + \frac{t}{2} \right] - (P_t)\left[ \frac{l_c}{2} + \frac{t}{2} \right]$$

$$= (6235.69)\left[ (125) + \frac{65}{2} + \frac{46}{2} \right] - (12471.38)\left[ \frac{65}{2} + \frac{46}{2} \right]$$

$$\boxed{(M_b)_h = 433.38 \times 10^3\ N\text{-}mm}$$

$$M_b = \sqrt{(M_b)_v^2 + (M_b)_h^2}$$

$$M_b = \sqrt{(738.97 \times 10^3)^2 + (433.38 \times 10^3)^2}$$

$$\boxed{M_b = 856.68 \times 10^3\ N\text{-}mm}$$

$M_t = P_t \cdot r$

$= (12471.38)(75)$

$$\boxed{M_t = 935.36 \times 10^3 \text{ N-mm}}$$

using Maxm shear stress theory

$$d_{s_1}^3 = \frac{16}{\pi \tau_{max}} \sqrt{(M_b)^2 + (M_t)^2}$$

$$d_{s_1}^3 = \frac{16}{\pi (40)} \sqrt{(856.68 \times 10^3)^2 + (935.38 \times 10^3)^2}$$

$d_{s_1} = 54.46$ mm

or

$$\boxed{d_{s_1} = 55 \text{ mm}}$$

(E) <u>Design of Right hand crank web:-</u>

on web

1. Bending stress due to $(R_2)_v$ in vertical plane
2. —— " —— " —— $P_t$ —— " Horizontal plane
3. Direct ~~shear~~ Compression stress due to $P_r$
4. Torsional shear stress due to $(R_1)_h$ & $P_t$

~~$(M_b)_r = (R_1)_v \left[b_1 + \frac{l_c}{2} + \frac{t}{2}\right]$~~

$$(M_b)_r = (R_2)_v \cdot \left[b_2 - \frac{l_c}{2} - \frac{t}{2}\right]$$

$$= (10632.7)\left[(125) - \frac{65}{2} - \frac{46}{2}\right]$$

$$\boxed{(M_b)_r = 738.97 \times 10^3 \text{ N-mm}}$$

$$(\sigma_b)_r = \frac{(M_b)_r \cdot (t/2)}{\frac{1}{12} w t^3}$$

$$(\sigma_b)_r = \frac{6 (M_b)_r}{w t^2}$$

~~738.97×10³ = 6[~~

$$(\sigma_b)_r = \frac{6 \times [738.97 \times 10^3]}{(78)(46)^2}$$

$$\boxed{(\sigma_b)_r = 27.94 \ N/mm^2}$$

$$\cancel{(M_b)_h = (R_2)_h \left[b_2 - \frac{l_{c1}}{2} - \frac{t}{2}\right]}$$

$$(M_b)_t = (P_t)\left[r - \frac{d_{s1}}{2}\right]$$

$$= (12471.38)\left[75 - \frac{55}{2}\right]$$

$$\boxed{(M_b)_t = 592.39 \times 10^3 \ N\text{-}mm}$$

$$(\sigma_b)_t = \frac{(M_b)_t (r)}{I}$$

$$(\sigma_b)_t = \frac{6 (M_b)_t}{t w^2}$$

$$(\sigma_b)_t = \frac{(592.39 \times 10^3)(6)}{(46)(75)^2}$$

$$\boxed{(\sigma_b)_t = 13.74 \ N/mm^2}$$

direct ~~compressive~~ compressive stress

$$(\sigma c)_d = \frac{P}{2wt}$$

$$(\sigma c)_d = \frac{21265.41}{2(75)(46)}$$

$$\boxed{(\sigma c)_d = 3.08 \text{ N/mm}^2}$$

$$\sigma c = (\sigma c)_t + (\sigma c)_r + (\sigma c)_d = 44.78 \text{ N/mm}^2 \quad \boxed{\sigma_c = 44.76 \text{ N/mm}^2}$$

Torsional stress

$$M_t = (R_2)_x \left[ b_2 - \frac{l_c}{2} \right]$$

$$= (6235.69)\left[125 - \frac{65}{2}\right]$$

$$\boxed{M_t = 576.80 \times 10^3 \text{ N-mm}}$$

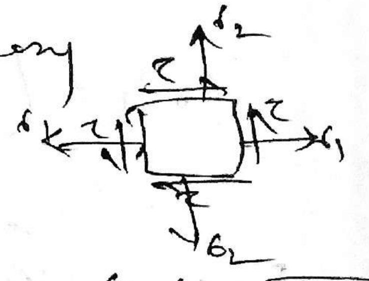
$$\tau = \frac{M_t r}{J}$$

$$= \frac{4.5 \, M_t}{w t^2}$$

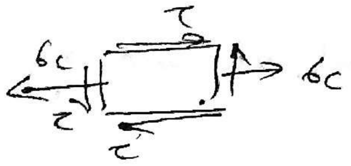
$$= \frac{(4.5)\left[576.80 \times 10^3\right]}{(75)(46)^2}$$

$$\boxed{\tau = 16.38 \text{ N/mm}^2}$$

Using ~~maximum~~ principal stress theory




$$(\sigma c)_{max} = \frac{\sigma_1 - \sigma_2}{2} \pm \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$




$$(\sigma_c)_{max} = \frac{\sigma_c}{2} + \sqrt{\left(\frac{\sigma_c}{2}\right)^2 - \tau^2}$$

$$= \frac{\sigma_c}{2} + \sqrt{\frac{\sigma_c^2}{4} - \tau^2}$$

$$= \frac{\sigma_c}{2} + \frac{1}{2}\sqrt{\sigma_c^2 - 4\tau^2}$$

$$= \frac{1}{2}\left[\sigma_c + \sqrt{\sigma_c^2 - 4\tau^2}\right]$$

$$= \frac{1}{2}\left[(44.76) + \sqrt{(44.76)^2 - 4(10.36)^2}\right]$$

$$\boxed{(\sigma_c)_{max} = 50.10 \text{ N/mm}^2}$$

The $(\sigma_c)_{max}$ is less than $(\sigma_c)_{all} = 75$ N/mm² hence design is safe.

(F) Design of left hand crank web:-

~~The~~ left hand crank web is identical to Right hand web.

(G) Design of crankshaft bearing:-

Bearing 2 subjected to maximum stress.

$$R_2 = \sqrt{\left[(R_2)_v + (R_2')_v\right]^2 + \left[(R_2)_h + (R_2')_h\right]^2}$$

$$= \sqrt{(10632.73 + 500)^2 + (6235.69 + 1000)^2}$$

$$\boxed{R_2 = 13277.53 \text{ N}}$$

diameter of journal at bearing 2 $(d_{s_1})$

$$\boxed{d_{s_1} = 55\,mm}$$

assuming $\frac{l}{d} = 1$

$$\boxed{l_2 = d_{s_1} = 55\,mm}$$

$$P_b = \frac{R_2}{d_{s_1} l_2}$$

$$= \frac{13277.53}{(55)(55)}$$

$$\boxed{P_b = 4.39\,N/mm^2}$$

$P_b < 10\,N/mm^2$

design is safe.

* side crankshaft at T.D.C Position

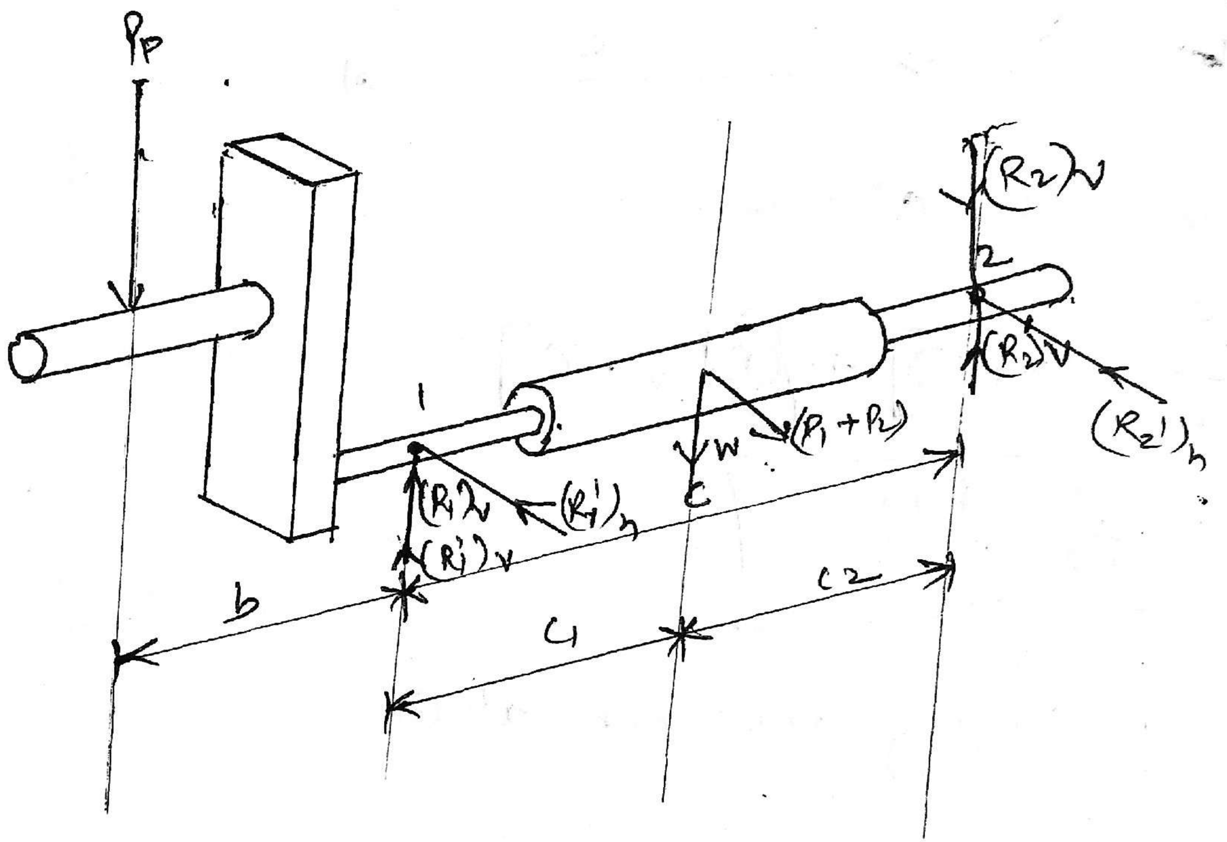
Assumption

① Engine is vertical & crank at T.D.C

② Belt drive is horizontal.

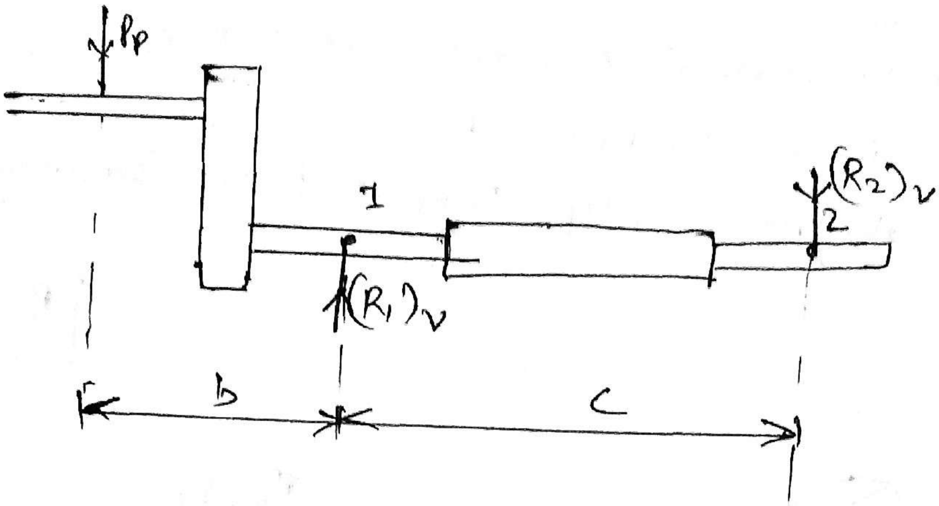
③ Crankshaft simply supported on Bearing 1 & 2

$$\boxed{P_p = (P_{max})\frac{\pi}{4}}$$



(A) Reactions :-

(1) when only load $P_p$ considered.




Taking moment at 2.

$$\Sigma M_2 = 0$$

$$0 = -[P_p \cdot (b+c)] + [(R_1)_v \cdot c]$$

$$\boxed{(R_1)_v = \frac{P_p(b+c)}{c}}$$

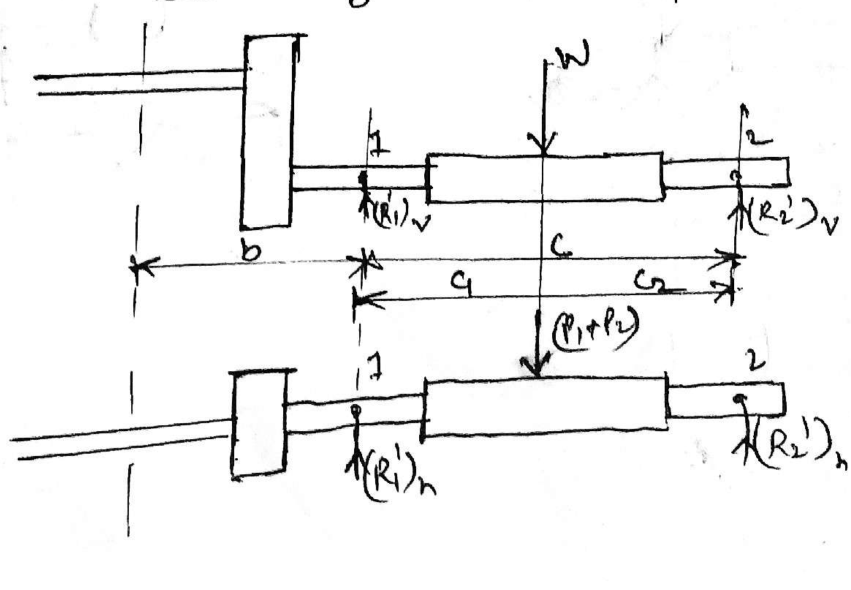
- Similarly taking moment at (1)

$$\Sigma M_1 = 0$$

$$0 = -[P_p(b)] + [(R_2)_v \cdot c]$$

$$\boxed{(R_2)_v = \frac{P_p \cdot b}{c}}$$

(2) when ~~bo~~ weight & pulley load ~~taken~~ only :-



In vertical plane.

Taking moment at 1

$$\Sigma M_1 = 0 = -[(R_2')_v \cdot C] + [W \cdot C_1]$$

$$\boxed{(R_2')_v = \frac{W \cdot C_1}{C}}$$

Taking moment at 2

$$\Sigma M_2 = 0 = [(R_1')_v \cdot C] - [W \cdot C_2]$$

$$\boxed{(R_1')_v = \frac{W \cdot C_2}{C}}$$

In horizontal plane,

Taking moment at 1

$$\Sigma M_1 = 0 = [(P_1 + P_2) \cdot C_1] - [(R_2')_h \cdot C]$$

$$\boxed{(R_2')_h = \frac{(P_1 + P_2) C_1}{C}}$$

Taking moment at 2

$$\Sigma M_2 = 0 = [(R_1')_h \cdot C] - [(P_1 + P_2) C_2]$$

$$\boxed{(R_1')_h = \frac{(P_1 + P_2) C_2}{C}}$$

(B) Design of Crankpin:-

1. Bearing Consideration
2. Bending Consideration.

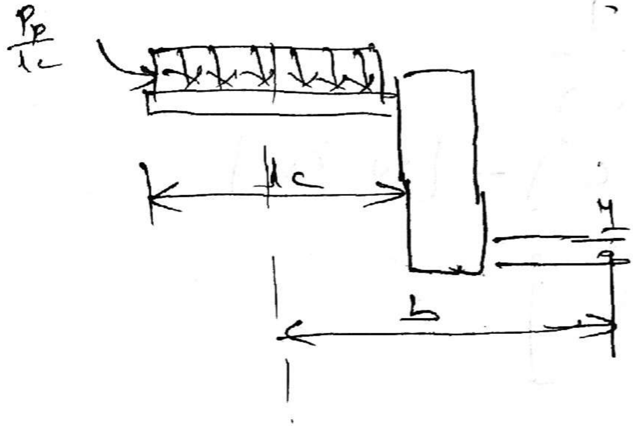
1. Bearing consideration

$$\boxed{(P_b)_c = \frac{P_p}{l_c \cdot d_c}}$$

(a) Generally $\frac{l_c}{d_c} = 0.6$ to $1.4$

(b) $(P_b)_c = 10$ to $12\ N/mm^2$

(2) Bending consideration

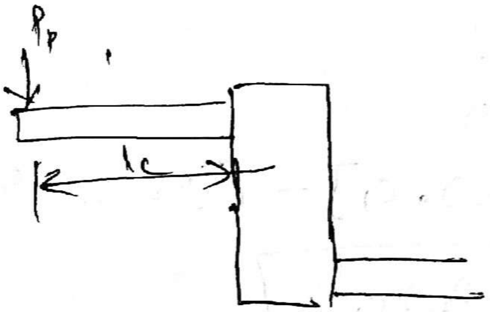



Assuming a UDL is acting on crank pin.

$P_p$

$\frac{l_c}{2}$

$$\boxed{M_b = P_p \cdot \frac{l_c}{2}}$$



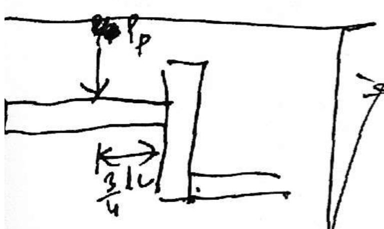

Similarly assuming the load is acting at end for maxm B.M,

$$\boxed{M_b = P_p \cdot l_c}$$

Taking value in between (mean value)

$$\boxed{M_b = \frac{3}{4}(P_p) \cdot l_c}$$

$$\sigma_b = \frac{M_b \cdot Y}{I}$$




$$= \frac{\frac{3}{4}(P_p) l_c \cdot \frac{d_c}{2}}{\frac{\pi}{64} d_c^4}$$

16

$$= \frac{32 (P_p) l_c}{\pi d_c^3}$$

$$\sigma_b = \frac{M_b \left(\frac{d_c}{2}\right)}{\frac{\pi}{64} d_c^4}$$

$$\boxed{\sigma_b = \frac{32 M_b}{\pi d_c^3}}$$

$d_c =$

$\frac{3}{4} \times \frac{1}{2}$

$\frac{3}{8} \times 64$

(c) Design of Bearings:-

$d_1$ = diameter of Journal or shaft at bearing 1

$l_1$ = length of bearing 1.

$\sigma_b$ = allowable bearing stress for shaft at bearing.

$t$ = thickness of web.

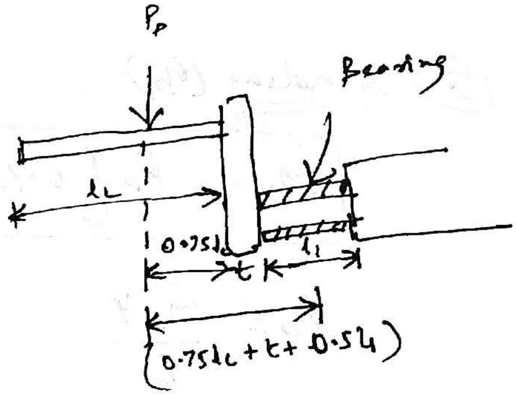
thickness of web (t),

$$\boxed{t = 0.45 d_c \text{ to } 0.75 d_c}$$

(I) By Bending Consideration:-

Bending moment at bearing 1.

$$\boxed{M_b = [0.75 l_c + t + 0.5 l_1] P_p}$$




By empirical relationship

$$\boxed{l_1 = 1.5 d_c \text{ to } 2 d_c}$$

$$\sigma_b = \frac{M_b y}{I}$$

$$\sigma_b = \frac{(M_b)\frac{d_1}{2}}{\frac{\pi}{64} d_1^4}$$

$$\boxed{\sigma_b = \frac{32 M_b}{d_1^3}}$$

(II) By Bearing Consideration

$$\boxed{P_b = \frac{R_1}{d_1 l_1}}$$

Take $P_b$ = 10 to 12 N/mm²

$$R_1 = \sqrt{[(R_1)_v + (R_1')_v]^2 + [(R_1')_h]^2}$$

Bearing 1 & 2 are identical.

(D) Design of crank web :-

There are two stresses in crank web:-

(I) Bending due to $P_p$

(II) Direct compressive due to $P_p$

(I) Bending ($\sigma_b$)

$$\boxed{M_b = P_p\left[0.75\, l_c + \frac{t}{2}\right]}$$

$$\sigma_b = \frac{M_b\, y}{I}$$

$$= \frac{M_b\left(\frac{t}{2}\right)}{\frac{1}{12} w t^3}$$

$$\boxed{\sigma_b = \frac{6 M_b}{w t^2}}$$

(II) Direct compressive ($\sigma_c$)

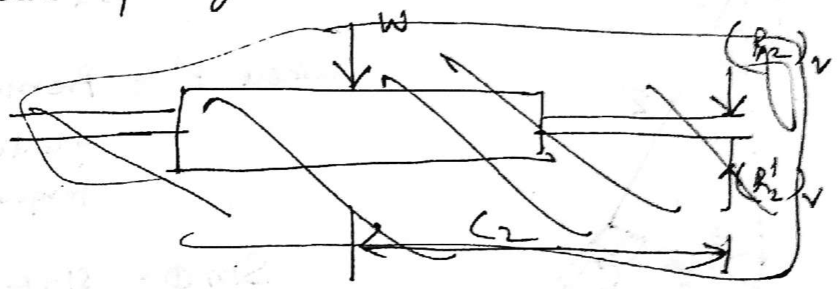
$$\boxed{\sigma_c = \frac{P_p}{w t}}$$

Total stress $(\sigma_c)_t$

$$\boxed{(\sigma_c)_t = \sigma_b + \sigma_c}$$

(E) Design of Shaft Under Flywheel:-

$d_s$ = dia. of shaft under Flywheel.




Bending moment in V.P:-

$$(M_b)_v = -P_p\,[b + c_1] + [(R_1)_v + (R_1')_v]\cdot c_1$$

Bending moment in H.P:-

$$(M_b)_h = (R_1')_h \cdot c_1$$

$$M_b = \sqrt{(M_b)_v^2 + (M_b)_h^2}$$

$$\sigma_b = \frac{M_b\, y}{I}$$

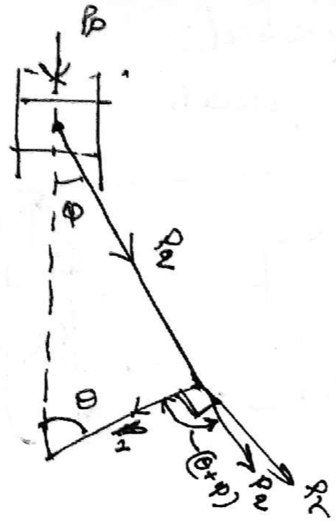
$$\sigma_b = \frac{M_b \cdot \frac{d_s}{2}}{\frac{\pi}{64} d_s^4}$$

$$\boxed{\sigma_b = \frac{32 M_b}{\pi d_s^3}}$$

Case II: Design of Side Crankshaft at Maximum torque angle:-

Assumptions:-

1. Engine is vertical
2. Belt drive is horizontal.
3. Crankshaft simply supported on bearing 1 & 2.




$$P_p = P'\left[\frac{\pi}{4}D^2\right]$$

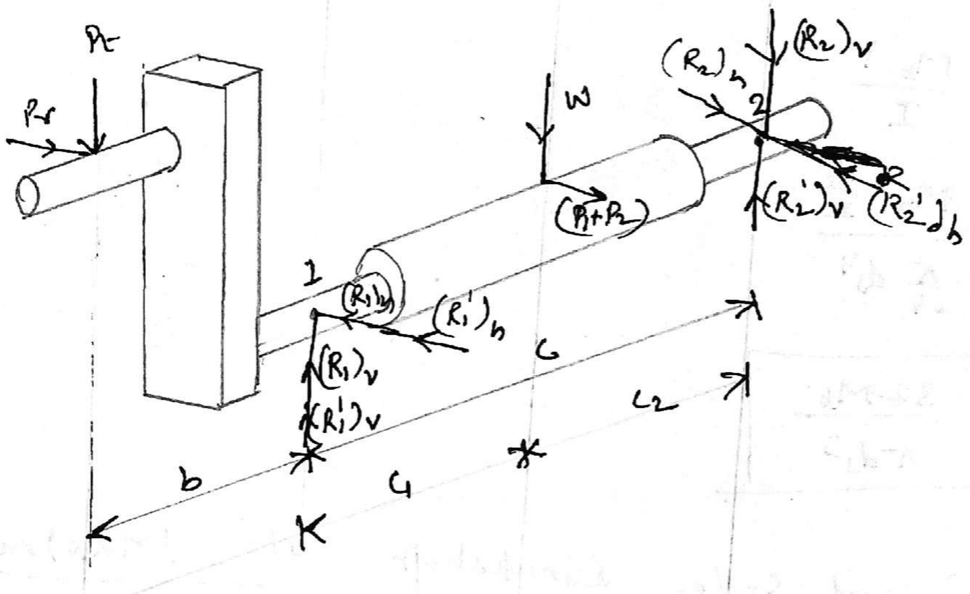
where $P'$ = Pressure in cylinder at max^m torque condn.

$$\sin\phi = \frac{\sin\theta}{n}$$

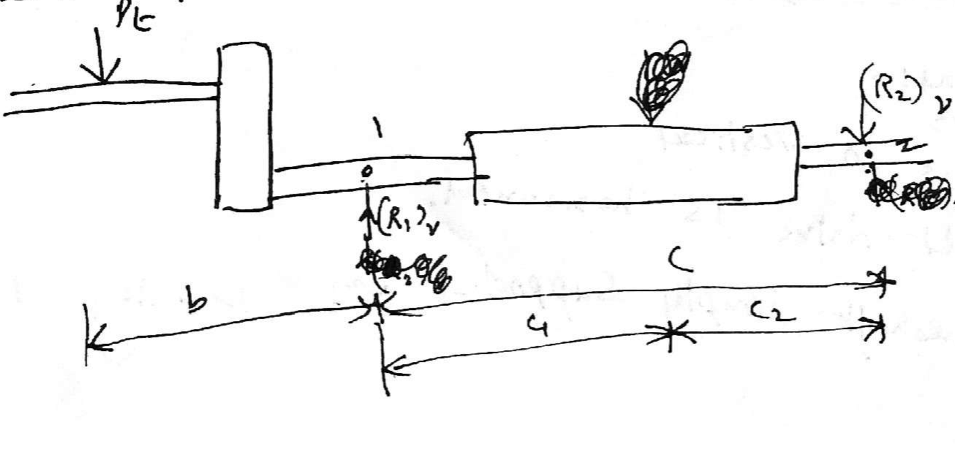
$$\cos\phi = \frac{1}{n}\sqrt{n^2-\sin^2\theta}$$

$$\boxed{\begin{aligned} P_t &= P_q \sin(\theta+\phi) \\ P_r &= P_q \cos(\theta+\phi) \end{aligned}}$$

(A) Bearing reactions




(1) when only load $P_t$ & $P_r$ considered



(a) In V.P

$$\Sigma M_1 = 0 = (-P_t \cdot b) + [(R_2)_v \cdot c]$$

$$P_t \cdot b = (R_2)_v \cdot c$$

~~Re = cde~~

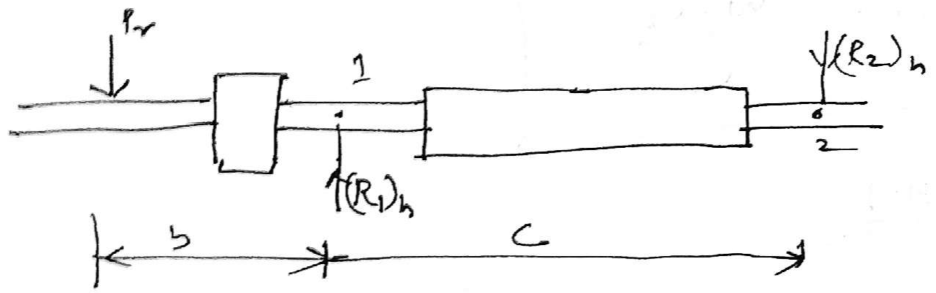
$$\boxed{(R_2)_v = \frac{P_t \cdot b}{c}}$$

$$\Sigma M_2 = 0$$

$$= -[P_t \cdot (b+c)] + [(R_1)_v \cdot c]$$

$$\boxed{(R_1)_v = \frac{P_t (b+c)}{c}}$$

(b) In. H.P




$$\Sigma M_1 = 0$$

$$0 = -(P_r \cdot b) + (R_2)_h \cdot c$$

$$\boxed{(R_2)_h = \frac{P_r \cdot b}{c}}$$

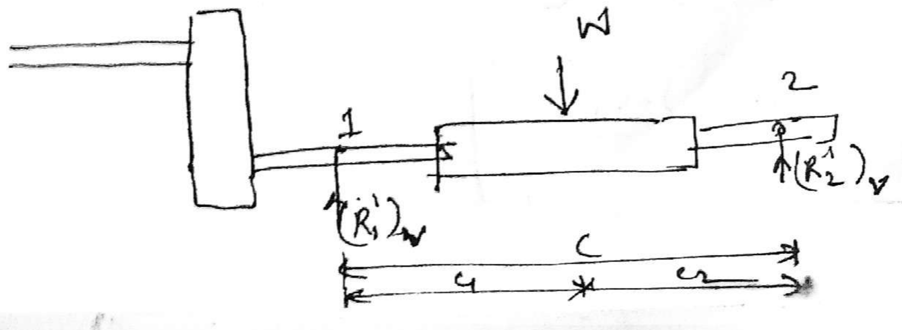
$$\Sigma M_2 = 0$$

$$0 = -[P_r \cdot (b+c)] + [(R_1)_h \cdot c]$$

$$\boxed{(R_1)_h = \frac{P_r \cdot (b+c)}{c}}$$

(2) When W & $(P_1 + P_2)$ are considered :-

(a) In V.P



$$\Sigma M_1 = 0$$

$$0 = -\left[(R_2')_v \cdot c\right] + \left[W \cdot c_1\right]$$

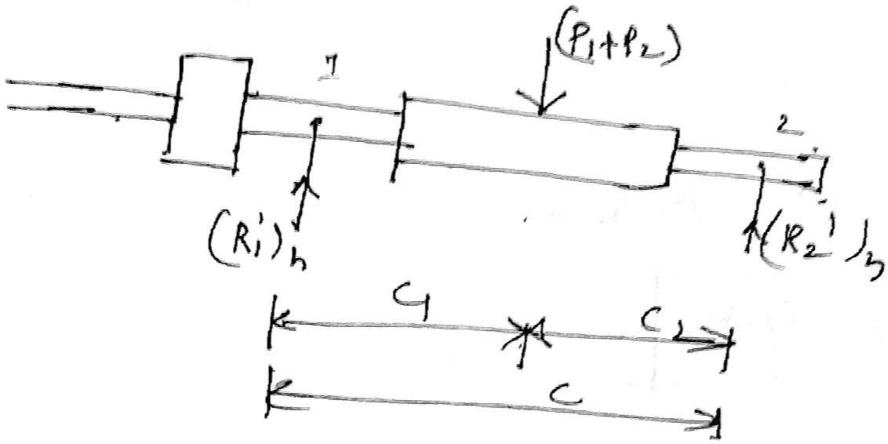
$$\boxed{(R_2')_v = \frac{W \cdot c_1}{c}}$$

$$\Sigma M_2 = 0$$

$$0 = \left[(R_1')_v \cdot c\right] - \left[W \cdot c_2\right]$$

$$\boxed{(R_1')_v = \frac{W \cdot c_2}{c}}$$

(b) In H.P




$$\Sigma M_1 = 0$$

$$0 = -\left[(R_2')_h \cdot c\right] + \left[(P_1+P_2) \cdot c_1\right]$$

$$\boxed{(R_2')_h = \frac{(P_1+P_2)\, c_1}{c}}$$

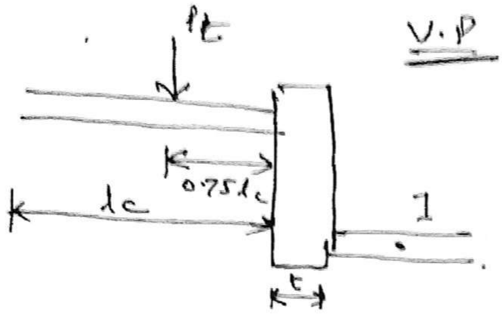
$$\Sigma M_2 = 0$$

$$0 = \left[(R_1')_h \cdot c\right] - \left[(P_1+P_2) \cdot c_2\right]$$

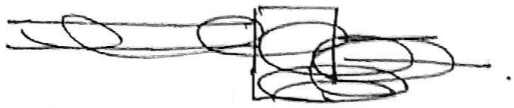
$$\boxed{(R_1')_h = \frac{(P_1+P_2) \cdot c_2}{c}}$$

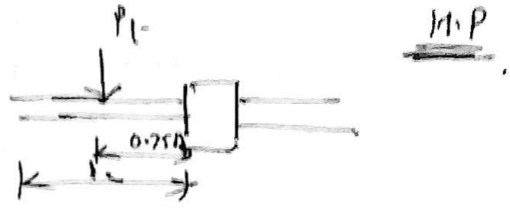
## (B) Design of Crank Pin

We know in Crank Pin load maxm load a distance of 0.75 lc from Crank web.




$$\boxed{(M_b)_v = P_t \cdot (0.75\, l_c)}$$






$$\boxed{(M_b)_h = P_t\,(0.75\, l_c)}$$

$$M_b = \sqrt{(M_b)_v^2 + (M_b)_h^2}$$

$$\boxed{\sigma_b = \frac{32 M_b}{\pi d_c^3}}$$

check for Crank Pin

## (C) Design of Crank web :-

Stresses :-

1. Direct compressive stress due to $P_r$
2. Torsional shear stress due to $P_t$
3. Bending stress due to $P_r$ & $P_t$

(1) Direct compressive stress :-

$$\boxed{(\sigma_c)_d = \frac{P_r}{wt}}$$

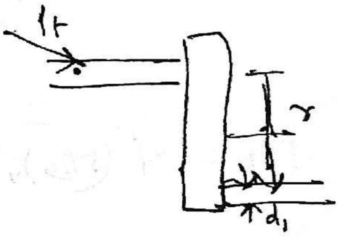
② Bending ~~Torsional~~ stress due to $P_r$ and $P_t$.

$$(M_b)_r = P_r \cdot \left[0.75 l_c + 0.5 t\right]$$

$$(\sigma_b)_r = \frac{(M_b)_r \cdot \left[\frac{t}{2}\right]}{\frac{1}{12} w t^3}$$

$$\boxed{(\sigma_b)_r = \frac{6 (M_b)_r}{w t^2}}$$

$$\cancel{(M_b)_t = P_t (0.75 l_c + 0.5)}$$




$$(M_b)_t = \left[r - \frac{d_1}{2}\right]$$

$$(\sigma_b)_t = \frac{(M_b)_t \cdot \left(\frac{w}{2}\right)}{\frac{1}{12} t w^3}$$

$$\boxed{(\sigma_b)_t = \frac{6 (M_b)_t}{t w^2}}$$

$$\boxed{\sigma_c = \sigma_{cd} + (\sigma_b)_t + (\sigma_b)_r} \longrightarrow ①$$

③ Torsional moment.

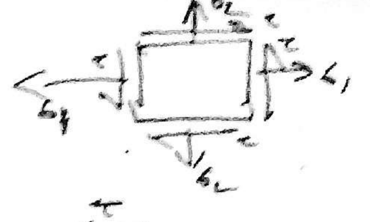
$$M_t = P_t \left(0.75 l_c + \frac{t}{2}\right)$$

$$\tau = \frac{M_t \cdot r}{J}$$

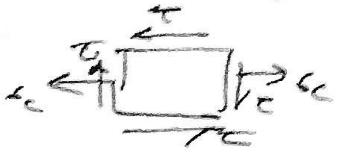
$$\boxed{\tau = \frac{4.5 M_t}{w t}} \longrightarrow ②$$

So from equtn ① & ② we have to use maxm Principle Stress theory max Principle stress theory,

$$\sigma_{max} = \frac{(\sigma_1 - \sigma_2)}{2} \pm \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$



$$= \left(\frac{\sigma_c}{2}\right) + \sqrt{\left(\frac{\sigma_c}{2}\right)^2 + \tau^2}$$



our case

$$\boxed{\sigma_{max} = \left(\frac{\sigma_c}{2}\right) + \frac{1}{2}\sqrt{\sigma_c^2 + 4\tau^2}}$$

(D) Design of shaft at Juncture of crank web:-

$d_{s1}$ = dia. of shaft at juncture of crank web.

Moments

(1) Bending moment in V.P due to $P_r$

(2) —"— —"— —"— H.P —"— $P_t$

(3) Torsional moment due to $P_t$

(1) B.M in V.P due to $P_r$

$$\boxed{(M_b)_v = P_r\,[0.75\,l_c + t]}$$

(2) B.M. in H.P due to $P_t$.

$$\boxed{(M_b)_h = P_t.\,[0.75\,l_c + t.]}$$

(3) T.M in due to $P_t$

$$\boxed{(M_t) = P_t\,.\,r.}$$

Resultant B.M.

$$M_b = \sqrt{(M_b)_v^2 + (M_b)_h^2}$$

Now there is $\sigma_b$ & $\tau$ so using

Maxm shear stress theory

$$\boxed{\tau_{max} = \frac{16}{\pi d_c^3}\sqrt{M_b^2 + M_t^2}}$$

(E) Design of shaft under Flywheel:-

$d_s$ = dia. of shaft under Flywheel.

max. B.M is acting at central plane.

$$(M_b)_v = -P_r\left[b + c_1\right] + \left[(R_1)_v + (R_1')_v\right] \cdot c_1$$

$$(M_b)_h = -P_t\,(b + c_1) + \left[(R_1)_h + (R_1')_h\right] \cdot c_1$$

$$\boxed{M_b = \sqrt{(M_b)_v^2 + (M_b)_h^2}}$$

$$\boxed{M_t = P_t \cdot r.}$$

Using max shear stress theory

$$\boxed{\tau_{max} = \frac{16}{\pi d_s^3}\sqrt{M_t^2 + M_b^2}}$$